Replacement and Reputation

Navin Kartik Elliot Lipnowksi Harry Pei

Motivation

Primary instrument to control elected officials: replacement

Replacement serves two roles:

1. Provide incentives
2. Selection

Other settings: organizational leaders, bureaucrats, etc.

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Our question: Does replacement lead to good outcomes in the long run?

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Simple model of accountability with moral hazard and adverse selection

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- Replacement is voters' only tool

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Takeaways

- Mechanism for and robustness of good long run outcomes varies
- In "incentive-sufficient" environs, can discipline moral hazard, but norms matter
  $\rightarrow$ despite reputational forces, poor long-run outcomes possible
  $\rightarrow$ novel rationale
- In "incentive-deficient" environs, discipline fails, but selection prevails

Related Literature

Political accountability with selection

- Banks & Sundaram (1993); Fearon (1999); Myerson (2006); Anesi & Buisseret (2022)

Reputation with imperfect monitoring

- Fudenberg & Levine (1992); Cripps, Mailath, Samuelson (2004)

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Reputation with exogenous replacement

- Mailath & Samuelson (2001); Tadelis (2002); Ekmekci, Gossner, Wilson (2012)

Reputation with competition between long-lived players

- Hörner (2002); Atakan & Ekmekci (2015); Deb & Fanning (2025)

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Time: $t = 0, 1, \ldots$

Pool of infinitely many identical long-lived politicians

Sequence of short-lived voters, one in each period

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In period 0, one politician is exogenously the incumbent

Incumbent chooses effort $a_0 \in \{0, 1\}$ and generates public signal $s_0 \sim f(\cdot|a_0) \in \Delta S$,

- Finite S; $f(\cdot|0) \neq f(\cdot|1)$ and both pmfs have full support

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Replacements are public; once a politician is replaced, he never returns

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- Good type exerts effort (action 1) whenever he is in office
- Opportunistic type's stage payoff is

$$u_s = \begin{cases} 0 & \text{if not in office in period } s \\ u(a_s) & \text{if in office in period } s, \end{cases}$$

where $u(0) \equiv 1 > u(1) \equiv 1 - \kappa > 0$. ($\kappa$ is effort cost)

Discount factor δ

Solution Concept

Equilibrium: symmetric weak PBE, i.e., weak PBE in which

- All opportunistic politicians use the same strategy:
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- Different politicians using different strategies

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  $\rightarrow$ all eqa are "good"
- But off-path flexibility plays role in other direction
  $\rightarrow$ construction of a "bad" eqm

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Proposition

A symmetric PBE exists.

Main Result

(Eventual) Full Effort

Definition

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Note: negative result also under weaker notions of eventual full effort

Equivalence Theorem

If an eqm attains FE, it has no learning; so FE must also be attainable absent good type

Intuitively, this requires conjunction of

low-enough effort cost; sufficiently-informative monitoring; enough patience

Formally on next slide → Condition FE-I

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So, always some eqm that attains EFE

But there is a tension between

- Eventual FE in all eqa
- FE (or even FE in first period) in some eqm

Condition FE-I

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There is a vector $(v(s))_{s\in S} \in [0,1]^S$ such that

$$(1-\delta)(1-\kappa) + \delta \sum_{s\in S} f(s|1)v(s) \geq (1-\delta)1 + \delta \sum_{s\in S} f(s|0)v(s) \qquad (\mathsf{IC_{FE}})$$

and

$$(1-\delta)(1-\kappa) + \delta \sum_{s\in S} f(s|1)v(s) \geq \max_{s\in S} v(s). \qquad (\mathsf{PK_{FE}})$$

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$$(1-\delta)(1-\kappa) + \delta \sum_{s\in S} f(s|1)v(s) \geq \max_{s\in S} v(s). \qquad (\mathsf{PK_{FE}})$$

Nec and suff for FE eqm

Holds if and only if

- κ is small enough; *and*
- f is sufficiently informative; *and*
- δ is large enough

What Prevents Good Long-Run Outcomes?

Theorem says some eqa do not attain Eventual FE when FE-I holds

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Let $\pi(h^t)$ denote the incumbent's reputation at history h^t. (Probability of good type.)

Proposition

In any eqm that does not attain Eventual FE,

$\exists$ pos-prob history h^t at which
$\pi(h^t) > \pi_0$ and yet incumbent is replaced with pos prob.

$\Longrightarrow$ replacement despite positive track record
($\because$ correctly expect lower effort than from new officeholder)

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Contrast with failure of Eventual FE in Myerson (2006)

- There, replacement cost is too high and there is never replacement

(EFEproof)

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Let π_t denote probability that period-t incumbent is a good type

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In every eqm:

1. Every opportunistic type is eventually replaced with probability 1.
2. If FE-I fails, every good type is retained forever with positive probability.

Hence, if FE-I fails, then with probability 1, some good type is never replaced.

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Logic for the first part is related to CMS (2004), "impermanent reputations"

(EFEproof)

Proofs Snippets

Plan

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I will discuss:

- not (1) $\implies$ not (3)
- (1) $\implies$ (3)

Omit:

- (1) $\implies$ (2)
- (2) [hence (4)] $\implies$ (1)
- (1) $\implies$ (4)

Equilibria that Fail Eventual FE

Why is it not easy to construct eqa that fail Eventual FE?

Naive attempt:

- opportunistic incumbent never works
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Solution when Condition FE-I holds: An eqm in which

- there isn't too much learning
- partial shirking in first period
- every incumbent gets replaced with prob 1

Hence Eventual FE violated

▸ Construction

FE-I Fails $\Longrightarrow$ Eventual FE in All Equilibria

- When Condition FE-I fails, cannot incentivize full effort—in particular, not in first period $\Longrightarrow$ there has to be some learning

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- $\forall \varepsilon > 0$, there is a pos prob that a new incumbent will get to belief above $1 - \varepsilon$
  - Loose intuition: If $\sup$[on-path beliefs] < 1, then incumbent would have to be working when near that $\sup$, contrary to FE-I not holding
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These arguments also lead to the two Propositions stated earlier. ▸ TrackRecord ▸ Selection

Conclusion

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A model of accountability with moral hazard and adverse selection

- Each politician is either good or opportunistic
- Replacement is instrument to provide incentives and to select good politicians

Takeaways

- Replacement makes long-run good outcomes always possible (in some eqm) and, sometimes guarantees it (in all eqa)
- Mechanism for good long-run outcomes varies
- In incentive-sufficient environs, can discipline moral hazard, but requires good norms
  $\rightarrow$ novel source of long-run inefficiencies: replacement despite good track record
- In incentive-deficient environs, discipline fails, but selection necessarily prevails

Appendix

Condition FE-I Fails $\implies$ $\mathbb{P}(a_0 = 1) < 1$ in every eqm

If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$

(Back)

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If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$ (Back)

Consider any $\varepsilon > 0$ small enough.

Because FE-I fails, there exists $s_0 \in S$ such that $V(s_0) \geq V(\emptyset) + \varepsilon$.

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- Voter optimality $\implies$ the opportunistic type must again be incentivized to play $a_1 = 1$
- There exists $s_1 \in S$ such that $V(s_0, s_1) \geq V(s_0) + \varepsilon$.

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Iterating, for any $t \geq 0$, there is a pos-prob history $(s_0, \ldots, s_t)$ s.t. $V(s_0, \ldots, s_t) \geq V(\emptyset) + t\varepsilon$

Contradiction, as $V(\cdot) \leq 1$

FE-I $\implies$ Equilibria that Attain FE

Consider the following strategy profile

- Incumbent always exerts effort on the eqm path.
- Voter t replaces the incumbent if and only signal s_{t-1} has a LR below some threshold.
- Off path, voter never retains incumbent and incumbent always shirks.

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We show that FE-I $\implies$ there is a threshold s.t. this profile is an eqm.

- Only thing to check is incumbent's incentive

Lemma

Assume Condition FE-I. There exist $\overline{v} > 0$ and $x \in (0, 1]$ such that (IC_{FE}) holds and

$$(1-\delta)(1-\kappa) + \delta \sum_{s \in S} f(s|1) v(s) = \overline{v},$$

with $v(s) = \overline{v} \cdot \mathbf{1}\left\{ \frac{f(s|1)}{f(s|0)} \geq x \right\}$

Incentive Lemma when FE-I Fails

Lemma

If Condition FE-I fails, then $\exists \varepsilon > 0$ s.t. $\forall \left(v(s)\right)_{s \in S} \in [0,1]^S$ that satisfies ($\mathsf{IC_{FE}}$),

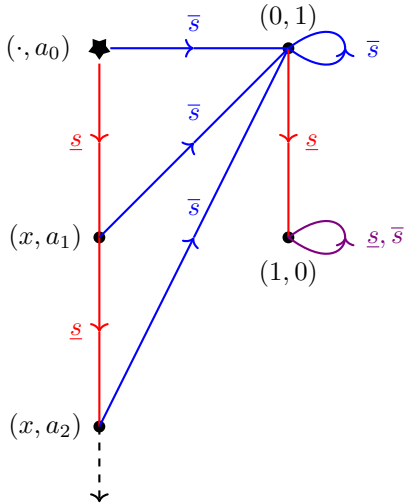
$$\max_{s \in S} v(s) \geq (1-\delta)(1-\kappa) + \delta \sum_{s \in S} f(s|1) v(s) + \varepsilon.$$

FE-I $\implies$ Equilibria that Fail Eventual FE

Construction with two signals, $S = \{\underline{s}, \overline{s}\}$.

FE-I $\implies$ Equilibria that Fail Eventual FE

Construction with two signals, $S = \{\underline{s}, \overline{s}\}$. The first number in the vector is the prob of replacement, and the second number is the prob of choosing $a = 1$. It holds that $0 < a_0 < a_1 < \cdots < 1$ and $x \in (0, 1]$.



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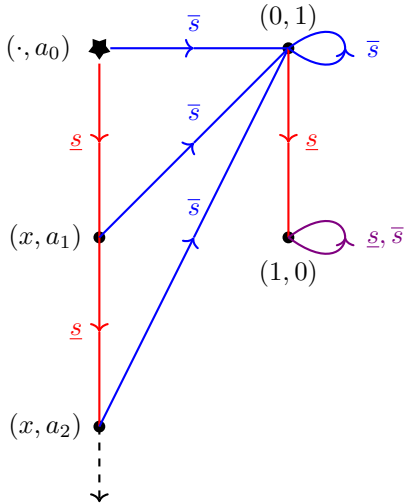

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Caveat:
use weak PBE to sustain $(1,0)$ profiles