Lemonade from Lemons: Information Design and Adverse Selection

Navin Kartik Weijie Zhong

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Introduction

- Adverse selection can limit efficient trade (Akerlof 70)
- Various mechanisms may help
  - signaling (Spence 73)
  - screening (Rothschild Stiglitz 76)
- Our paper: information design
  - fix an interdependent-values TIOLI environment
  - characterize **all** outcomes achievable by (only) varying participants' info
- Interpretations
  - an info designer with some objective (e.g., seller)
  - predictions across info structures

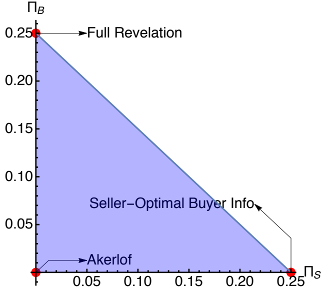
Example

Example (1)

- Seller with one indivisible good
- Seller's valuation: $q \in \{0,1\}$; uniform distr
- Buyer's valuation: $v(q) = (3/2)q$; makes TIOLI offer $p \in \mathbb{R}$
- Seller, Buyer Payoffs: $\begin{cases} (0,0) & \text{if no trade} \\ (p-q, v(q)-p) & \text{if trade} \end{cases}$
- (Akerlof benchmark) Informed seller; Uninformed buyer
  - eqm price $p \leq 0$; no gains from trade; foregone surplus of $1/4$

Example (2)

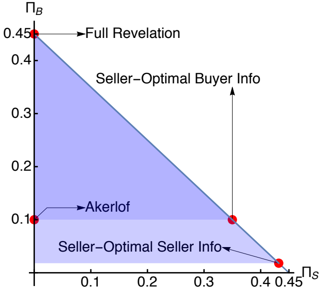
- Informed seller; Uninformed buyer: no gains from trade



- Both informed: eqm price $p = q$; all surplus to buyer
- Is there buyer info (w/ informed seller) giving all surplus to seller?
  - Yes: signal $s \in \{0, 1\}$; $\Pr(s = 1|q = 1) = 1,\ \Pr(s = 1|q = 0) = 1/2$
- **Any** point in the triangle is implementable with suitable buyer info
- Nothing else can be implemented with *any* (even joint) info design

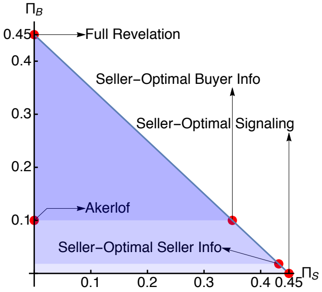
Example (3)

- Now suppose $v(q) = (3/2)q + \mathbf{1/5}$
  - Akerlof benchmark: $p = 0$; still inefficient, but some gains from trade



- Implement more using seller info design, with uninformed buyer
  - e.g., signal $s \in \{0,1\}$: $\Pr(s=1|q=1)=1$; $\Pr(s=1|q=0)=\varepsilon$ small $\varepsilon \implies p=0$ but less trade $\implies \Pi_B \downarrow$ from Akerlof
- Outcomes implementable with uninformed buyer (and seller info design) is a superset of fully-informed seller (and buyer info design)

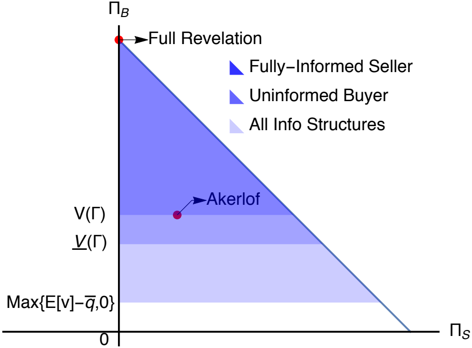
Example (4)



- Can implement still more with superior-informed buyer
  - e.g., uninformed seller; buyer signal $s \in \{0,1\}$: $\Pr(s=1|q=1)=\varepsilon$, $\Pr(s=1|q=0)=0$; buyer offers $p=\mathbb{E}[v]=0.95$; seller gets all the surplus

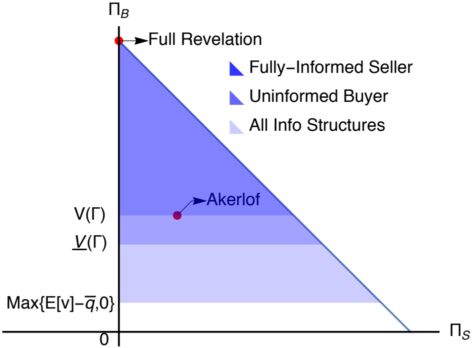
    nb: signaling, but only off-path
- Using suitable joint info design (and signaling), can fill in the triangle

General Results



- Uninformed buyer enough if no signaling / superior-informed seller
- Triangles coincide if and only if either
  - Akerlof buyer is willing to offer highest price
  - Akerlof eqm has no gains from trade
- Only in these—special but salient—cases, no need to reduce seller info

Literature



Info design for monopoly pricing

- Bergemann, Brooks, & Morris 2015
- Roesler & Szentes 2017

More broadly related

- Kessler 2001; Levin 2001
- Kamenica & Gentzkow 2011

Model

Model (1)

- Two players decide on a joint action (trade)
- "Seller" valuation: $q \in Q \equiv [\underline{q}, \overline{q}] \subset \mathbb{R}$; prior distr μ
- "Buyer" valuation: measurable function $v(q) \geq q$
- Private signals $t_b, t_s \sim f(t_b, t_s|q)$: info structure; design variable
- Buyer makes TIOLI offer $p \in \mathbb{R}$
- Seller, Buyer vNM payoffs: $\begin{cases} (0,0) & \text{if no trade} \\ (p-q, v(q)-p) & \text{if trade} \end{cases}$
- "Perfect Bayesian Equilibrium"
  - seller sells iff $\mathbb{E}[q|p, t_s] \leq p$
  - buyer best responds with pricing strategy $\sigma(p|t_b)$
  - beliefs satisfy Bayes Rule + No Signaling What You Don't Know

    seller forms belief about $t_b|p, t_s$ satisfying Bayes on path; both on- and off-path, updates about q only based on t_s and belief about t_b

Model (2)

- Seller, Buyer payoffs: $\begin{cases} (0,0) & \text{if no trade} \\ (p-q, v(q)-p) & \text{if trade} \end{cases}$
- Buyer makes TIOLI offer $p \in \mathbb{R}$

Comments:

1. Not assuming $v(\cdot) \uparrow$, only $v(q) \geq q$
2. Besides adverse selection, also subsume
   - Monopoly pricing (flip labels): $Q \subset \mathbb{R}_-$ and $v(q) = 0$; trade at $p \in \mathbb{R}_-$
     - $t_s = q$ and t_b informative: 3rd degree price discrim
   - Advantageous selection / negative correlation:
     - Common $u \geq 0$ from joint project; "B" demands transfer t from "S"
       $\rightarrow$ seller's payoff is $(\underbrace{-t}_{p}) - (\underbrace{-u}_{q})$ and buyer's is $\underbrace{u}_{v(q)=-q} - (\underbrace{-t}_{p})$

Results

Preliminaries

Let $\Gamma \equiv (v, \mu)$ be an environment and $\mathcal{T}$ an information structure

An eqm has no signaling if seller does not update about quality from price (on- and off-path)

- assured if, e.g., S's signal is a sufficient statistic for q: $f(t_s, t_b|q) = f(t_b|t_s)f(t_s|q)$

Define

$$R(\Gamma, \mathcal{T}) \equiv \{(\Pi_S, \Pi_B) : \exists \text{ an eqm under } \mathcal{T} \text{ with these payoffs}\}$$

$$R(\Gamma) \equiv \bigcup_{\mathcal{T}} R(\Gamma, \mathcal{T})$$

$$R_{ns}(\Gamma) \equiv \text{subset of } R(\Gamma) \text{ achieved with no signaling}$$

Let $\mathbf{T}_S$ be set of uninformed-buyer info structures
Let $\mathbf{T}_B$ be set of fully-informed seller info structures

- any eqm given $\mathcal{T} \in \mathbf{T}_S \cup \mathbf{T}_B$ has no signaling

Uninformed Buyer (1)

$$U(\Gamma) \equiv \int_Q (v(q) - q)\mu(q)\mathrm{d}q \qquad \text{surplus}$$

$$V(\Gamma, \mathcal{T}) \equiv \text{ buyer payoff given } \mathcal{T} \in \mathbf{T}_S$$

$$\underline{V}(\Gamma) \equiv \inf_{\mathcal{T} \in \mathbf{T}_S} V(\Gamma, \mathcal{T}) \qquad \text{min payoff for uninformed buyer}$$

Theorem

$$\mathbf{R}_{ns}(\Gamma) = \bigcup_{\mathcal{T} \in \mathbf{T}_S} R(\Gamma, \mathcal{T}) = \left\{ (\Pi_S, \Pi_B) : \begin{array}{l} \Pi_S \geq 0 \\ \Pi_S + \Pi_B \leq U(\Gamma) \\ \Pi_B \geq \underline{V}(\Gamma) \end{array} \right\}.$$

- Implication: w/o signaling, sufficient to focus on uninformed buyer
- Can replace no signaling with either buyer option to be uninformed, or with power to price as function (specifically, indep) of her signal

Uninformed Buyer (2)

Proof sketch:

- Immediate that $\Pi_S \geq 0$ and $\Pi_S + \Pi_B \leq U(\Gamma)$
- With no signaling, $\Pi_B \geq \underline{V}(\Gamma)$
- Show that any such (Π_S, Π_B) is implementable with B uninformed
  1. $(0, U(\Gamma))$ is implementable by making S uninformed
  2. Given any implementable (Π_S, Π_B), and any $\Pi'_B \in [\Pi_B, U(\Gamma)]$, some (Π'_S, Π'_B) is implementable by randomizing seller's info suitably

     $\implies$ set of no-signaling buyer payoffs is interval $[\underline{V}(\Gamma), U(\Gamma)]$
  3. Use key garbling lemma below

Lemma

$\forall \mathcal{T} \in \mathbf{T}_S$, $\forall \Pi_S \in [0, U(\Gamma) - V(\Gamma, \mathcal{T})]$, $\exists \widetilde{\mathcal{T}} \in \mathbf{T}_S$ *such that*

$$(\Pi_S, V(\Gamma, \mathcal{T})) \in R(\Gamma, \mathcal{T}).$$

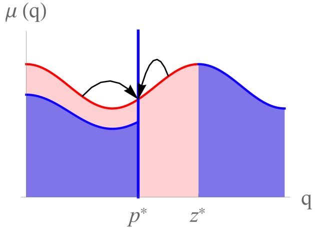
Uninformed Buyer (3)

Lemma

$\forall \mathcal{T} \in \mathbf{T}_S$, $\forall \Pi_S \in [0, U(\Gamma) - V(\Gamma, \mathcal{T})]$, $\exists \widetilde{\mathcal{T}} \in \mathbf{T}_S$ *such that*

$$(\Pi_S, V(\Gamma, \mathcal{T})) \in R(\Gamma, \mathcal{T}).$$

Suppose $\mathcal{T}$ has seller fully informed and μ is atomless:



Garble $\mathcal{T}$ so that buyer trades with all $q < z^*$ at price p^*

- $z^* \leftarrow$ surplus: $\Pr(q < z^*)\mathbb{E}[v(q) - q | q < z^*] = V(\Gamma, \mathcal{T}) + \Pi_S$
- $p^* \leftarrow$ buyer payoff: $\Pr(q < z^*)\mathbb{E}[v(q) - p^* | q < z^*] = V(\Gamma, \mathcal{T})$

Fully-Informed Seller (1)

$\mathbf{R}_B(\Gamma) \equiv$ subset of $\mathbf{R}_{ns}(\Gamma)$ achieved with fully-informed seller

Theorem

$$\mathbf{R}_B(\Gamma) = \left\{ (\Pi_S, \Pi_B) : \begin{array}{l} \Pi_S \geq 0 \\ \Pi_S + \Pi_B \leq U(\Gamma) \\ \Pi_B \geq V(\Gamma, \mathcal{T}_{\mathit{Akerlof}}) \end{array} \right\}.$$

- Similar ideas as Bergemann, Brooks & Morris (2015)
- Can also get "unique implementation"

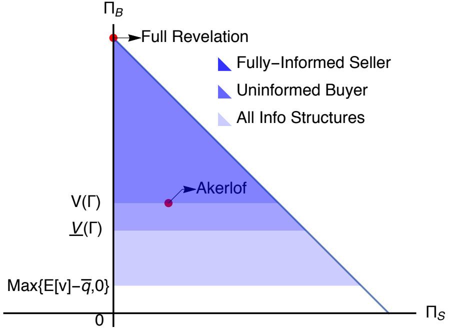
Signaling

Theorem

$$\mathrm{cl}(\mathbf{R}(\Gamma)) = \left\{ (\Pi_S, \Pi_B) : \begin{array}{l} \Pi_S \geq 0 \\ \Pi_S + \Pi_B \leq U(\Gamma) \\ \Pi_B \geq \max\{\mathbb{E}_\mu[v(q)] - \bar{q}, 0\} \end{array} \right\}.$$

- When $\mathbb{E}_\mu[v(q)] \geq \bar{q}$, how to get full trade with price $\bar{q}$?
  - uninformed seller; buyer is informed with prob ε only when $q \in B_\varepsilon(\bar{q})$
  - buyer offers $\bar{q}$; any $p < \bar{q}$ is rejected $\because$ seller infers $q \approx \bar{q}$
- This constr satisfies D1 refinement if (and only if) $\mathbb{E}_\mu[v(q)] \geq v(\bar{q})$
  - sufficient that $v(\cdot)$ is non-$\uparrow$: e.g., monopoly pricing and adv selection
- But constr can be augmented to satisfy D1 more generally
  - seller also receives info
- Different kind of constr to achieve $\Pi_S = 0$ while minimizing Π_B

In Sum



Further issues

- characterizing uninformed-buyer info structures (✓ linear v)
- more on signaling (monotonicity / other refinements)
- other mechanisms
  - if $\mathbb{E}[v] - \bar{q} < 0$, cannot implement any more s.t. participation
    mech design w/ Akerlof info cannot substitute for info design
  - if $\mathbb{E}[v] - \bar{q} > 0$, mech design is useful