Lemonade from Lemons: Information Design and Adverse Selection

Navin Kartik   Weijie Zhong

December 2019

Introduction

- Asymmetric information can affect market outcomes
  - adverse selection can limit efficiency (Akerlof 70; Stiglitz Weiss 81)
  - private information can alter distribution (Mussa Rosen 78)

- Various mechanisms alter—help or hurt—outcomes
  - signaling (Spence 73)
  - screening (Rothschild Stiglitz 76)

- Our paper: information design
  - fix a canonical interdependent-values trading environment
  - characterize **all** outcomes as participants' info varies

    $\rightarrow$ interested in more than just efficiency

- Interpretations
  - designer with some objective (e.g., buyer or regulator)
  - predictions across info structures

Punchlines

- Information design can achieve a lot
  - with no restrictions, all feasible and "indiv. rational" payoffs
  - restrictions to canonical classes of info **do** matter;
    but in some salient cases, do not

    $\rightarrow$ The (non)limits to adverse selection

- Methodological contributions
  - allow information to vary on both sides of market
  - identify role of canonical information classes

Example

Example (1)

- Seller can sell one indivisible good

	Prob$(1/2)$	Prob$(1/2)$
Buyer's valuation v	1	2
Seller's cost $c(v)$	$1/2$	2

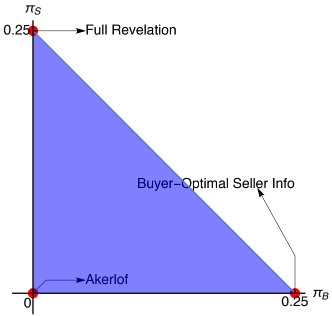
- Seller posts a TIOLI price $p \in \mathbb{R}$

- Payoffs:

	Seller	Buyer
No trade	0	0
Trade	$p - c(v)$	$v - p$

- Akerlof benchmark: Fully-informed Buyer; Uninformed Seller
  - eqm price $p \geq 2$; no gains from trade; foregone surplus $1/4$

Example (2)

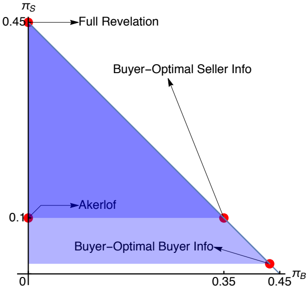


- Both informed: eqm price $p = v$; all surplus to Seller
- Is there Seller info (w/ informed Buyer) giving all surplus to Buyer?
  - Yes: reveal $c = 2$ sometimes and o-wise induce belief with $\mathbb{E}c = 1$. Upon latter, Seller prices at 1, efficient trade, no surplus to Seller.
- **All** points in $\triangle$ obtain w/ some Seller info (and informed Buyer)
- Feasibility $+$ IR $\implies$ nothing else implemented with **any** info design

Example (3)

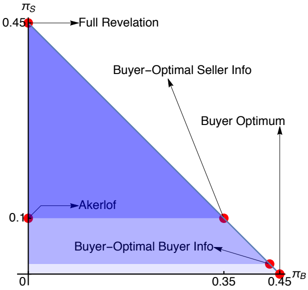
	Prob$(1/2)$	Prob$(1/2)$
Buyer's valuation v	1	2
Seller's cost $c(v)$	$\mathbf{0.3}$	$\mathbf{1.8}$

- Akerlof benchmark: $p = 2$; still inefficient, but some gains from trade



- Implement other payoffs with Buyer info design, and uninformed Seller
- In fact, a superset of those with fully-informed Buyer

Example (4)

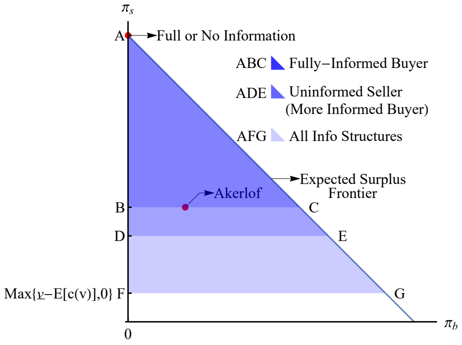


- No more can be implemented if Buyer is more-informed than Seller
- But can implement still more otherwise
  - e.g., Uninformed Buyer; with ε pr. Seller is informed of $v = 1$

    Seller's $p = \mathbb{E}c$ indep of signal; Buyer gets all the surplus

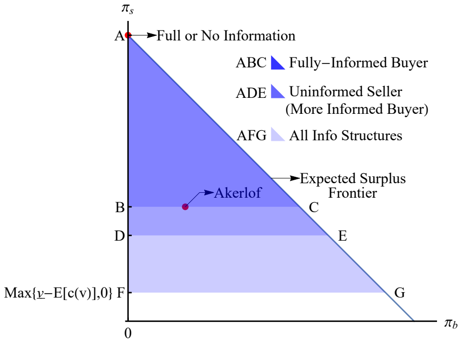
    $\rightarrow$ Seller's info makes off-path belief credible
- Using joint info design, can fill in the entire feasible & IR $\triangle$

General Results



- Uninformed Seller sufficient for more-informed Buyer
  - more generally, if Buyer does not update from price
- All three triangles coincide if and only if either
  - Akerlof info can generate full trade
  - Akerlof info can generate no trade

Literature



Monopoly pricing

- Bergemann, Brooks, & Morris 2015
- Roesler & Szentes 2017

...

Info design in games

- Bergemann & Morris 2016
- Makris & Renou 2019

...

Others

- Kessler 2001; Levin 2001
- Bar-Isaac, Jewitt, Leaver 2018

...

Model

Model

- Buyer's valuation: $v \in [\underline{v}, \overline{v}]$; prior μ with support V
- Seller's cost: $c(v) \leq v$, continuous with $\mathbb{E}[v - c(v)] > 0$
- Private signals $t_b, t_s \sim P(t_b, t_s | v)$: info structure; design variable
  $\rightarrow$ private signals are wlog
- Seller posts a price $p \in \mathbb{R}$; Buyer decides whether to accept
- Seller, Buyer vNM payoffs: $\begin{cases} (0,0) & \text{if no trade} \\ (p - c(v), v - p) & \text{if trade} \end{cases}$
- weak Perfect Bayesian Equilibrium
  - $+$ strengthenings

Nb: not assuming $c(v) \uparrow$

subsumes monopoly pricing, adverse or favorable selection

Canonical Info Structures and Notation

$\Gamma \equiv (c(v), \mu)$ is the environment

Canonical classes of information structures

- $\mathbf{T}$: all (joint) info structures
- $\mathbf{T}_{mb}$: Buyer more informed than Seller, i.e., t_b is suff statistic for v
- $\mathbf{T}_{us}$: Seller uninformed
- $\mathbf{T}_{fb}$: Buyer fully-informed

Implementable payoffs

- $\mathbf{\Pi}(\Gamma)$: payoff vectors across all info structures and all wPBE
- $\mathbf{\Pi}^*(\Gamma)$: subset with price-independent beliefs
  $\rightarrow$ Buyer does not update from price, after conditioning on t_b
  $\rightarrow$ implied by NSWYDK if Buyer more informed
- $\mathbf{\Pi}_i^*(\Gamma)$: further subset when information structure is restricted to class $i = mb, us, fb$

$$\mathbf{\Pi}_{us}^*(\Gamma) \cup \mathbf{\Pi}_{fb}^*(\Gamma) \subset \mathbf{\Pi}_{mb}^*(\Gamma) \subset \mathbf{\Pi}^*(\Gamma) \subset \mathbf{\Pi}(\Gamma)$$

Results

All Info Structures

Total surplus: $\mathbb{E}[v - c(v)] \equiv S(\Gamma)$

Seller guarantee: $\max\left\{\underline{v} - \mathbb{E}\left[c(v)\right], 0\right\} \equiv \underline{\pi}_s(\Gamma)$

Buyer guarantee: 0

Theorem

Consider all information structures and equilibria.

$$\mathbf{\Pi}(\Gamma) = \left\{ (\pi_b, \pi_s) : \begin{array}{l} \pi_b \geq 0 \\ \pi_s \geq \underline{\pi}_s(\Gamma) \\ \pi_b + \pi_s \leq S(\Gamma) \end{array} \right\}.$$

Moreover, $\forall \varepsilon > 0$ $\exists$ a finite information structure and price grid whose set of **sequential—even "D1"—equilibrium** *payoffs is an ε-net of $\mathbf{\Pi}(\Gamma)$.*

Nb: a single information structure implements entire payoff set

More-informed Buyer

$$\underline{\pi}_s^{us}(\Gamma) \equiv \inf \{\pi_s : \exists(\pi_b, \pi_s) \in \mathbf{\Pi}^*_{us}(\Gamma)\}$$

Theorem

Consider equilibria with price-independent beliefs.

1. $\mathbf{\Pi}^*(\Gamma) = \mathbf{\Pi}^*_{mb}(\Gamma) = \mathbf{\Pi}^*_{us}(\Gamma)$.
2. $\mathbf{\Pi}^*_{us}(\Gamma) = \{(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma) : \pi_s \geq \underline{\pi}_s^{us}(\Gamma)\}$.
3. $\forall(\pi_b, \pi_s) \in \mathbf{\Pi}^*_{us}(\Gamma)$ *with* $\pi_s > \underline{\pi}_s^{us}(\Gamma)$, $\exists \tau \in \mathbf{T}_{us}$ *s.t. all equilibria have payoffs* (π_b, π_s).

- Given price-indep beliefs, uninformed Seller is sufficient
- Only additional constraint now is $\underline{\pi}_s^{us}(\Gamma) \geq \underline{\pi}_s(\Gamma)$. Inequality is strict if $\underline{v} \leq \mathbb{E}[c(v)]$ and $c(v) < v \ \forall v$.
- Unique implementation

Proof Sketch

Fully-Informed Buyer

$$\underline{\pi}_s^{fb}(\Gamma) \equiv \sup_p \int_p^{\bar{v}} (p - c(v))\mu(\underset{\cdot}{v})$$

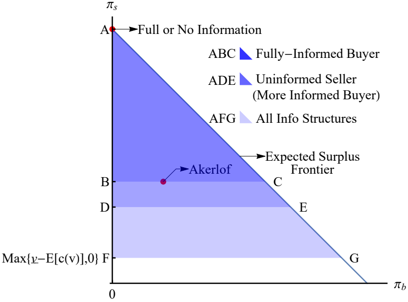
Theorem

Consider a fully informed Buyer and price-independent beliefs.

1. $\mathbf{\Pi}^*_{fb}(\Gamma) = \{(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma) : \pi_s \geq \underline{\pi}_s^{fb}(\Gamma)\}$.
2. $\forall (\pi_b, \pi_s) \in \mathbf{\Pi}^*_{fb}(\Gamma)$ *and* $\varepsilon > 0$, $\exists \tau \in \mathbf{T}_{fb}$ *with all eqm payoffs in* ε*-ngbhd of* (π_b, π_s).

- Of course, $\underline{\pi}_s^{fb}(\Gamma) \geq \underline{\pi}_s^{us}(\Gamma)$; strictly if $\underline{\pi}_s^{fb}(\Gamma) > \underline{\pi}_s(\Gamma)$
- Proof via "incentive compatible distributons", generalizing Bergemann, Brooks & Morris' (2015) "extreme markets"
- Approx. unique implementation

In Sum



Further issues

- characterizing uninformed-Seller bound $\underline{\pi}_s^{us}$ (✓ linear v)
- more general correlation in c, v (✓ if $c \leq v$)
- negative trading surplus (✓ for all info structures)
- other mechanisms
  - if $\underline{v} - \mathbb{E}[c(v)] \leq 0$, cannot implement any more s.t. participation
  - if $\underline{v} - \mathbb{E}[c(v)] > 0$, mech design is useful

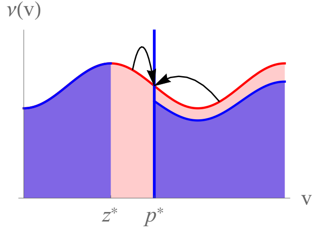
Appendix

Theorem 2: Proof Sketch

Lemma

$\forall(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma)$ with $\pi_s > \underline{\pi}_s^{us}(\Gamma)$, $\exists$ garbling of τ^* s.t. all equilibria have payoffs (π_b, π_s).

Suppose τ^* has fully-informed Buyer and prior μ has density:



Garble τ^* so that Seller posts price p^* and there is trade only with all $v > z^*$

- $z^* \leftarrow$ surplus: $\pi_s + \pi_b = \Pr(v > z^*)\mathbb{E}[v - c(v)|v > z^*]$
- $p^* \leftarrow$ Seller payoff: $\pi_s = \Pr(v > z^*)\mathbb{E}[p^* - c(v)|v > z^*]$

Back