Implementation with Evidence

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Introduction

- General (hidden information) implementation problem:

  Design a game form in which decentralized strategic behavior leads to desirable outcomes

- The design specifies
  - messages which can be sent to the planner
  - outcomes selected for each profile of messages sent
  - ...
- Typically, all messages are assumed to be cheap talk
  - Available to an agent in all states of the world
  - Don't directly affect payoffs
- **Restrictive and precludes interesting class of problems**

Introduction: Motivation

Example 1

Divide money among employees depending on their individual output

- If agents can only send cheap-talk messages, unrestrained manipulation
- But might be able to request verification of output
  - If an agent cannot show *more* output than actually produced, and showing any subset is costless

    $\Longrightarrow$ setting with hard/state-contingent evidence
  - If agents can borrow output at some cost and/or there is a cost of "carrying output to court"

    $\Longrightarrow$ setting with costly signaling

Introduction: Motivation

Example 2

Income taxation problem

- the planner cannot observe agents' income
- each agent has a document that stipulates her income
- planner requests agents to submit the document
  - an agent can either costlessly submit his true document

    or
  - fabricate a false document at some cost

$\implies$ setting with costly signaling/evidence fabrication

Introduction: Contribution

- state-contingent evidence / costly evidence fabrication is introduced into a standard implementation setting

Two main issues of interest:

- given some evidentiary structure, what social objectives can be fully implemented?
- given a social objective, what minimal evidentiary structure is needed for implementation?
  - step towards thinking about designing evidentiary structures

By-products:

- rank informativeness of evidentiary structures
- rank social objectives in ease of implementation

Introduction: Contribution

- revisit Maskin's (1999/1977) results in this more general setting
  - complete information
  - Nash implementation
  - allow for any mechanisms [incl. integer games]
- Maskin-monotonicity no longer generally necessary
- provide a (weaker) necessary condition that is also sufficient under usual conditions
- use this to study implications of evidentiary structures
- permissive results in contrast with standard negative results

Related Literature

- full implementation
  - hard evidence: Ben-Porath and Lipman (2009)
  - feasible implementation: Dagan, Serrano, Volij (1999), Hurwicz, Maskin, Postlewaite (1995)
- partial implementation
  - hard evidence: Green and Laffont (1986), Bull and Watson (2007), Deneckere and Severinov (2007), ...
  - costly evidence provision: Bull (2008)
- communication games
  - hard evidence: Milgrom (1981), Lipman and Seppi (1995), ... , Glazer and Rubinstein (2001/4/6), Sher (2008)
  - costly signaling & evidence fabrication: Spence (1973), ... , Kartik (2009)

Plan

Model: Basics

- Finite set of players, $I = \{1, \cdots, n\}$
- Set of outcomes / allocations, A $\quad (|A| > 1)$
- Set of states of nature, Θ $\quad (|\Theta| > 1)$
- Preferences for each player i are represented by

$$u_i : A \times \Theta \to \mathbb{R}$$

  Nb: Ordinal vs. vNM preferences

- A Social Choice Function (SCF) is $f : \Theta \to A$
  - paper deals with correspondences

Model: Evidence Structure

- In each state θ, agent i is endowed with a set of evidence, E_i^θ
  - document, receipt, legal record, verbal proof, collateral, ...

- Interpretation
  - at θ, i can provide any $e_i \in E_i^\theta$ costlessly
  - evidence is non-falsifiable

    $\implies e_i \notin E_i^\theta$ is not available at θ

    or

    fabricating evidence is prohibitively costly

    $\implies e_i \notin E_i^\theta$ is available at θ but infinitely costly

    - infinite cost is an approximation; relaxed later

Model: Evidence Structure

- Notation
  - $\mathcal{E} := \{E_i^\theta\}$ is an evidence structure
  - $E^\theta := E_1^\theta \times \cdots \times E_n^\theta$
  - $E_i := \bigcup_\theta E_i^\theta$
  - $E := E_1 \times \cdots \times E_n$
- e_i is cheap talk for agent i if and only if

$$e_i \in \bigcap_{\theta \in \Theta} E_i^\theta$$

- Special case: the standard setting without evidence

$$\forall i, \forall e_i \in E_i : e_i \text{ is cheap talk}$$

Model: Mechanisms and Implementation

- A mechanism is a pair (M, g) where
  - $M = M_1 \times \cdots \times M_n$ is the (cheap-talk) message space
  - $g : M \times E \to A$ is an outcome function
- A mechanism defines a strategic-form game in each state θ:
  - A pure strategy for player i is $(m_i, e_i) \in M_i \times E_i$
  - For each $(m, e) \in M \times E$, player i's payoff is $u_i(g(m, e), \theta)$
  - $NE(M, g, \theta)$ is the set of pure strategy Nash equilibria
- A mechanism (M, g) implements a SCF f if

$$\forall \theta : \; f(\theta) = \{a : a = g(m, e) \text{ for some } (m, e) \in NE(M, g, \theta)\}$$

Model: Comments

1. Evidence submission is inalienable, i.e. voluntary choice

   $\Longrightarrow$ cannot treat it as part of the allocation space

   $\Longrightarrow$ "moral hazard" aspect

2. Distinguish states from preference profiles

3. A planner can always "ignore" evidence

   $\Longrightarrow$ evidence can only broaden scope for implementation

4. Results extend to mixed NE if $\forall i, \theta$, $u_i(\cdot, \theta)$ is bounded

5. Without loss of generality:
   - Non-empty evidence sets
   - Planner knows evidence structure
   - Submit exactly one piece of evidence
   - Static mechanisms (under cost interpretation)

An Example

- $N = \{1, 2\}$
- $\Theta = \{\theta_1 < \cdots < \theta_K\}$
- $A = \mathbb{R}^2_+$, so $a = (a_1, a_2)$
- Preferences: $u_i(a, \theta) = u_i(a_i)$, str. increasing

Remark.
Without any evidence, a SCF f is implementable if and only if it is constant.

- Intuition: Full implementation and state-independent prefs
- Nb: true for any solution concept

An Example: Adding Evidence

- Suppose now that player 1 can provide evidence $e_1 \in E_1^\theta = \{\theta_1, \ldots, \theta\}$
  - $e_1 = \theta_1$ is available in all states, can be interpreted as silence
- Player 2 has no evidence: $E_2^\theta = E_2^{\theta'}$ for all θ, θ'
- Define

$$\mathcal{F} = \{f : range[f] \subseteq \mathbb{R}^2_{++}\}$$

An Example: Adding Evidence

Claim.

Any $f \in \mathcal{F}$ can be implemented with the given evidentiary structure.

Proof.

Let $M_2 = \Theta$ and use the outcome function

$$g(e_1, m_2) = \begin{cases} f(e_1) & \text{if } m_2 = e_1 \\ (\text{“}+\infty\text{”}, 0) & \text{if } m_2 < e_1 \\ (0, 0) & \text{if } m_2 > e_1 \end{cases}$$

□

- Simple and well-behaved mechanism
- Rationalizability is enough ($\Longrightarrow$ no bad MSNE)
- Agent 2's cheap-talk message is important

Plan

Maskin-Monotonicity: A Reminder

- $L_i(a,\theta) := \{b : u_i(a,\theta) \geq u_i(b,\theta)\}$
- Given some f, say that θ' is monotonically related to θ if

$$\forall i : L_i(f(\theta),\theta) \subseteq L_i(f(\theta),\theta')$$

- A SCF f is Maskin-monotonic if for all θ, θ',

$$\theta' \text{ monotonically related to } \theta \implies [f(\theta) = f(\theta')]$$

- Well-known that this can be a demanding requirment
  - Any monotonic SCF defined on unrestricted domain of preferences is constant (Saijo, 1987)
  - If preferences are state-independent, a monotonic SCF is constant
  - Nb: demanding even when we consider correspondences

Maskin-Monotonicity: A Reminder

Theorem (Maskin)

Without evidence, a SCF is implementable only if it is Maskin-monotonic.

Proof.

- Pick a mechanism (M, g) that implements f
- Take any θ' monotonically related to θ
- Pick any $s^* \in NE(M, g, \theta)$

$$\implies g(s^*) = f(\theta)$$

$$\implies s^* \in NE(M, g, \theta')$$

$$\implies f(\theta) = f(\theta')$$

$\square$

Maskin-Monotonicity

When evidence is available, this argument could fail because either

- $s^* = (m^*, e^*) \in NE(M, g, \theta)$ may not be available at θ'

  or

- the set of possible deviations may expand from θ to θ'

Indeed, Maskin-monotonicity is not necessary with evidence:

(Extreme) Example: $\forall \theta$, $E_1^{\theta} = \{\theta\}$. Any SCF is implementable.

Evidence-Monotonicity

S-EM

NS

Definition

f is Maskin-monotonic provided that $\forall \theta, \theta'$,

if

- θ' is monotonically related to θ

then $f(\theta) = f(\theta')$.

Evidence-Monotonicity

S-EM

NS

Definition

f is evidence-monotonic provided that $\forall\theta\ \exists e^*_\theta \in E^\theta$ s.t. $\forall\theta, \theta'$,

if

- θ' is monotonically related to θ

  and

- $e^*_\theta \in E^{\theta'}$ and $E^{\theta'} \subseteq E^\theta$

then $f(\theta) = f(\theta')$.

Remark.

Weaker then Maskin-monotonicity, coinciding if, and generally only if, there is "no evidence."

Evidence-Monotonicity: Example

Example 1

$\Theta = \mathbb{R}^n_+$. For any i and $\theta = (\theta_1, \cdots, \theta_n)$, $E_i^\theta = [0, \theta_i]$.

Claim: Any SCF is evidence-monotonic.

Proof: For each θ : set $e^*_\theta = \theta$.

Since $\forall \theta \neq \theta'$: $\exists i$ s.t. $\theta_i \neq \theta'_i$, two cases:

- $\theta_i > \theta'_i \Rightarrow \theta_i \notin E_i^{\theta'} \Rightarrow e^*_\theta \notin E^{\theta'}$
- $\theta_i < \theta'_i \Rightarrow \theta'_i \notin E_i^\theta \Rightarrow E^{\theta'} \nsubseteq E^\theta$

Evidence-Monotonicity: Example

Example 2

Dist

- Two propositions, a and b, each is true or false:

  $\Theta = \{\phi, a, b, ab\}$

- Experts can provide proof but due to time constraint:

  $E_i^{\phi} = \{\phi\}; E_i^{a} = \{\phi, a\}; E_i^{b} = \{\phi, b\}; E_i^{ab} = \{\phi, a, b\}$

- preferences are state independent
  (hence, any pair of states are monotonically related)

1. Suppose $f(ab) = f(b)$

   f is evidence-monotonic: set $e_{\phi}^{*} = \phi, e_{a}^{*} = a, e_{b}^{*} = e_{ab}^{*} = b$

2. Suppose $\tilde{f}(ab) \notin \{\tilde{f}(a), \tilde{f}(b)\}$

   $\tilde{f}$ is not evidence-monotonic

Evidence-Monotonicity: Necessity

THEOREM.
A SCF is implementable only if it is evidence-monotonic.

Proof.

- suppose (M, g) implements f
- for each θ, pick $s^*_\theta = (e^*_\theta, m^*_\theta) \in NE(M, g, \theta)$
- consider any θ' and θ s.t.
  - θ' is monotonically related to θ
  - $e^*_\theta \in E^{\theta'}$ and $E^{\theta'} \subseteq E^\theta$

    $\Leftrightarrow$ s^*_θ is feasible at θ' and no new deviations

$\implies s^*_\theta \in NE(M, g, \theta')$

$\implies f(\theta) = g(s^*_\theta) = f(\theta')$ $\square$

Strong Evidence-Monotonicity

- If a player who has extra deviations at θ' cannot gain by deviating, the same argument would still apply ▸ NS

Definition

f is strong evidence-monotonic provided that $\forall \theta\ \exists e^*_\theta \in E^\theta$ s.t. $\forall \theta, \theta'$,

if

- θ' is monotonically related to θ

  and

- $e^*_\theta \in E^{\theta'}$ and $[\forall i : E_i^{\theta'} \subseteq E_i^\theta$ or $f(\theta) \in \arg\max_b u_i(b, \theta')]$

then $f(\theta) = f(\theta')$. ▸ EM

Theorem

A SCF f is implementable only if it is strong evidence-monotonic.

Strong Evidence-Monotonicity: Sufficiency

Condition (No Veto Power)

$\forall \theta, a$: *if* $\left| \left\{ i : a \in \arg\max_{b \in A} u_i (b, \theta) \right\} \right| \geq n - 1$, *then* $a = f(\theta)$.

Nb: Moore & Repullo's (1988) *restricted* veto power would also do

- NVP is often mild when $n \geq 3$ (some examples later)

Theorem.

Assume NVP and $n \geq 3$.

A SCF is implementable if and only if it is strong evidence-monotonic.

Sufficiency: the mechanism

For each $i : M_i = \Theta \times A \times \mathbb{N}$

- **Rule 1:** If $m_1 = \cdots = m_n = (\theta, f(\theta), k)$ and $e = e^*_\theta$
  $\implies$ outcome is $f(\theta)$
- **Rule 2:** If for some i: $m_j = (\theta, f(\theta), k)$ and $e_j = e^*_{j,\theta}$ for all $j \neq i$ while $(m_i, e_i) = (\theta_i, b_i, k_i, e_i) \neq (\theta, f(\theta), k, e^*_{i,\theta})$
  - Case (a): $e_i \in E_i^\theta$

    $\implies$ outcome is b_i if $f(\theta) \succeq_{i,\theta} b_i$; outcome is $f(\theta)$ o-wise
  - Case (b): $e_i \notin E_i^\theta$

    $\implies$ pick the outcome announced by i
- **Rule 3:** For any other case

  $\implies$ outcome announced by player with highest integer

Sufficiency: proof

Assume θ' is the true state

- It is clear that "truthtelling" is an equilibrium i.e., $m_1 = \cdots = m_n = (\theta', f(\theta'), k)$ and $e = e^*_{\theta'}$ is an eqm.

Sufficiency: proof

Assume θ' is the true state. Pick any equilibrium.

<u>To show:</u> the outcome induced is $f(\theta')$

- if the equilibrium falls into Rule 3, the outcome must be each player's favorite, hence by NVP, must be $f(\theta')$
- similarly, if it falls into Rule 2, must be favorite of all players except possibly i, hence by NVP, must be $f(\theta')$
- so suppose the eqm falls into Rule 1, i.e.
  $m_1 = \cdots = m_n = (\theta, f(\theta), k)$ and $e = e^*_\theta$; outcome is $f(\theta)$
  - θ' is monotonically related to θ *(by Rule 2a)*
  - $e^*_\theta \in E^{\theta'}$ *(feasibility / prohibitive cost)*
  - $\forall i : E_i^{\theta'} \subseteq E_i^{\theta}$ or $f(\theta) \in \arg\max_b u_i(b, \theta')$ *(by Rule 2b)*

$\implies f(\theta) = f(\theta')$ by strong evidence-monotonicity $\square$

Sufficiency: Evidence-Monotonicity

- While Strong EM is sufficient (with NVP and $n \geq 3$), EM need not be
- Intuition: may not be able to give a player who can disprove others' lie the incentive to do so

▸ Example

▸ S-EM

Condition (Non-satiation)

$\forall i, \theta, \theta'$: $\exists a \in A$ s.t. $u_i(a, \theta') > u_i(f(\theta), \theta')$.

▸ Weaker

Remark.

If NS holds, Strong EM is equivalent to EM.

Corollary.

Assume $n \geq 3$, NVP, and NS. A SCF is implementable if and only if it is evidence-monotonic.

On NS and NVP

Although not universal, both NS and NVP are satisfied in many situations with $n \geq 3$. Roughly, require "enough disagreement":

- Economic environments (Moore and Repullo, 1988)
- Any environment where the planner can augment allocations with additional arbitrarily small transfers off-the-equilibrium path (cf. Sanver, 2006; Benoît and Ok, 2008; Ben-Porath and Lipman, 2009)
- Some pure public goods problems without transfers

Distinguishability

Useful to provide an alternative characterization of evidence-monotonicity.

Definition

A state $\theta \in \Theta$ is distinguishable from an event $\Omega \subseteq \Theta$ if

$$\forall \Omega' \subseteq \Omega : \bigcup_{\theta' \in \Omega'} E^{\theta'} \neq E^{\theta}.$$

- For every $\Omega' \subseteq \Omega$, either some player can disprove Ω' at θ, or some player can disprove θ at some $\theta' \in \Omega'$
- If θ is distinguishable from Ω, it is distinguishable from each subset of Ω
- In general, θ distinguishable from θ' & θ'' (pairwise) does not imply θ is distinguishable from $\{\theta', \theta''\}$

E.g.

Distinguishability

Definition

Given a SCF f and state θ, let $T^f(\theta)$ be the set of states θ' s.t.

$$\left[\theta' \text{ is monotonically related to } \theta\right] \text{ and } \left[f(\theta) \neq f(\theta')\right]$$

- $T^f(\theta)$ is the set of "problem states" (wrt $f(\theta)$) in the standard setting
- f is Maskin-monotonic if and only if

$$\bigcup_{\theta \in \Theta} T^f(\theta) = \emptyset$$

Distinguishability & Evidence-monotonicity

Proposition.
f is evidence-monotonic if and only if

$$\forall \theta : \theta \text{ is distinguishable from } T^f(\theta)$$

Distinguishability & Evidence-monotonicity

Proposition.
f is evidence-monotonic if and only if

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Remark.

- *Clear that θ must be distinguishable from each $\theta' \in T^f(\theta)$*
- *But generally more is needed (recall example)*
- *"Non problematic" state-event pairs are "monotonic-ized" via preferences rather than evidence*

$\implies$ *precisely what evidence structures allow implementation of a given SCF (under $n \geq 3$, NS, and NVP)*

Universal Distinguishability

Definition

The evidence structure satisfies universal distinguishability if

$$\forall \theta : \ \Omega \subseteq \Theta \diagdown \{\theta\} \implies \theta \text{ is distinguishable from } \Omega.$$

Universal Distinguishability

Definition

The evidence structure satisfies universal distinguishability if

$$\forall \theta: \ \Omega \subseteq \Theta \setminus \{\theta\} \implies \theta \text{ is distinguishable from } \Omega.$$

- Satisfied by various common assumptions in hard information communication literature (e.g., structures that lead to "unraveling results" à la Milgrom, 1981)

COROLLARY.

Assume $n \geq 3$ and universal distinguishability. Any SCF that satisfies NS and NVP can be implemented.

▸ Proof

Normal Evidence Structures

Lipman and Seppi (1995); Forges and Koessler (2005); Bull and Watson (2007):

Definition

An evidence structure satisfies normality if $\forall i, \theta$, $\exists \bar{e}_{i,\theta} \in E_i^\theta$ s.t.

$$\bar{e}_{i,\theta} \in E_i^{\theta'} \implies E_i^\theta \subseteq E_i^{\theta'}$$

Interpretation

- If at θ, player i cannot exclude θ' using $\bar{e}_{i,\theta}$, then no other available evidence for i can exclude θ'

  $\implies$ $\bar{e}_{i,\theta}$ is maximal: it proves by itself what i could prove by jointly sending all his available evidence at θ

- A setting with no time/space/effort constraints satisfies this, because any conjunction of evidence is also available

Normal Evidence Structures

Proposition.
Assume the evidence structure is normal. For any $\theta \in \Theta$ and $\Omega \subseteq \Theta$, if θ is distinguishable from each $\theta' \in \Omega$ then θ is distinguishable from Ω.

- Under normality, only need to check distinguishability pairwise
- Intuition: can focus on the "maximal" evidence profile in any state

Normal Evidence Structures

COROLLARY.

Assume $\mathcal{E}$ is normal. SCF f is evidence-monotonic if and only if

$$\forall\theta : \theta' \in T^f(\theta) \implies E^{\theta} \neq E^{\theta'}.$$

COROLLARY.

Assume normality, NS, NVP, and $n \geq 3$.

Any SCF is implementable if the evidence structure satisfies pairwise distinguishability:

$$\forall\theta, \theta' : E^{\theta} \neq E^{\theta'}$$

- Requires only that planner can distinguish between any pair of states if he had access to entire set of available evidence in each state

Ranking Evidence Structures

Can use the notion of distinguishability to partially order evidence structures.

Definition

$\tilde{\mathcal{E}}$ is more informative than $\mathcal{E}$, denoted $\tilde{\mathcal{E}} \blacktriangleright \mathcal{E}$, if any $\theta \in \Theta$ and $\Omega \subseteq \Theta$ that are distinguishable under $\mathcal{E}$ are also distinguishable under $\tilde{\mathcal{E}}$.

Nb: Universal distinguishability $\blacktriangleright \cdots \blacktriangleright$ no evidence

Ranking Evidence Structures

Proposition.
Assume that $\tilde{\mathcal{E}} \blacktriangleright \mathcal{E}$. If a SCF is evidence-monotonic under $\mathcal{E}$ it is also evidence-monotonic under $\tilde{\mathcal{E}}$.

Corollary.
Assume that $\tilde{\mathcal{E}} \blacktriangleright \mathcal{E}$ and $n \geq 3$. Let f be a SCF satisfying no veto power and non-satiation. If f is implementable under $\mathcal{E}$ then f is also implementable under $\tilde{\mathcal{E}}$.

Ranking Social Choice Functions

Can also use distinguishability to partially order SCFs.

Definition

f is more Maskin-monotonic than h, denoted $f \trianglerighteq h$, if

$$\forall\theta : T^f(\theta) \subseteq T^h(\theta).$$

Proposition.

If $f \trianglerighteq h$, then if h is evidence-monotonic under evidence structure $\mathcal{E}$, f is also evidence-monotonic under $\mathcal{E}$.

Corollary.

Assume $n \geq 3$ and f and h are SCFs satisfying NVP and NS such that $f \trianglerighteq h$. If h is implementable under $\mathcal{E}$, then f is also implementable under $\mathcal{E}$.

Introduction

Hard Evidence

Fabricable Evidence

Fabricable Evidence: Setting

- Hard evidence can be thought of as if the cost of sending $e_i \in E_i$ is either 0 (if $e_i \in E_i^\theta$) or $+\infty$ (if $e_i \notin E_i^\theta$)
- We now introduce an explicit richer cost function

$$c_i(e_i, \theta) \in \mathbb{R}_+ \cup \{+\infty\}$$

  and assume wlog

$$E_i^\theta = \{e_i \in E_i : c_i(e_i, \theta) = 0\} \neq \emptyset$$

- Fairly general costly signaling environment where preferences are given by

$$u_i(a, \theta) - c_i(e_i, \theta)$$

  - can in fact dispense with separability

- Notion of implementation: no costly evidence be sent at equilibrium

Cost-Monotonicity

Definition

f is cost-monotonic provided that $\forall\theta\ \exists e^*_\theta$ such that for any θ, θ', if

$$e^*_\theta \in E^{\theta'}$$

and

$$u_i(f(\theta),\theta) \geq u_i(a,\theta) - c_i(e_i,\theta) \Rightarrow u_i(f(\theta),\theta') \geq u_i(a,\theta') - c_i(e_i,\theta')$$

then $f(\theta) = f(\theta')$.

Remark.

If $c_i(\cdot,\cdot) \in \{0, +\infty\}$ ($\approx$ hard evidence), the above definition reduces to strong evidence-monotonicity.

Cost-Monotonicity

Theorem

A SCF is implementable only if it is cost-monotonic. ▸ Mechanism

With $n \geq 3$, a SCF satisfying NVP is also implementable if it is cost-monotonic.

Remark.

Any SCF is cost-monotonic if players have a "slight preference for honesty" (cf. Matsushima, 2008 and Dutta & Sen, 2009). Formally, for each player i, $E_i = \Theta$ and

$$c_i(\theta, \theta') = \begin{cases} 0 \text{ if } \theta = \theta' \\ \varepsilon \text{ if } \theta \neq \theta' \end{cases}$$

where $\varepsilon > 0$ can be arbitrarily small.

Sufficiency: Mechanism given Preferences for Honesty

For each i, $M_i = A \times \mathbb{N}$

- **Rule 1:** If for some i: $m_j = (a, k_j)$ and $e_j = \theta$ for all $j \neq i$ $\implies$ outcome is $f(\theta)$
- **Rule 2:** Otherwise, outcome announced by player with highest integer

Proof: Suppose true state is θ'.

1. "Truthtelling" is an eqm.
2. If $(n-1)$ or fewer agents produce $e_i = \theta$ with $a_i = a$, the associated outcome must be top-ranked by $n-1$ agents.
3. If n agents produce $e_i = \theta$ with $a_i = a$, no-one can change the outcome, so $\theta = \theta'$.

Bayesian Implementation

Conclusion

- Main message: hard evidence / costly signaling can dramatically increase scope for implementation
- Characterization uses complete information assumption substantially
- But the themes carry over to incomplete information
  - e.g. small preference for honesty mechanism readily extends
  - more generally, weakening of Jackson's (1992) Bayesian-monotonicity condition
- Our mechanisms use
  - Nash Equilibrium concept
  - integer games

  but the approach can be applied to other equilibrium concepts (e.g. Ben-Porath and Lipman, 2009) or, we hope, to "bounded" mechanisms (future work)

Thank you!

Evidence-Monotonicity: Insufficiency Example

Back

- $n = 4$. $\Theta = \{\theta_1, \theta_2\}$. $A = \{a, b\}$.
- $E_1^{\theta_1} = \{x\}$, $E_1^{\theta_2} = \{x, y\}$; for $i \neq 1$, $E_i^{\theta_1} = E_i^{\theta_2} = \{z\}$.
- For all θ and $i \in \{1, 2\} : u_i(b, \theta) > u_i(a, \theta)$.
  For all θ and $i \in \{3, 4\} : u_i(a, \theta) > u_i(b, \theta)$.
- $f(\theta_1) = b$ and $f(\theta_2) = a$.
- f is EM: $e^*_{\theta_1} = (xzzz)$ and $e^*_{\theta_2} = (yzzz)$
- NVP trivially satisfied (3 players never agree on top-ranked)
- But f is not implementable
  - there must exist $s^* \in NE(M, g, \theta_1)$ s.t. $f(s^*) = b$
  - players 3 and 4 cannot unilaterally deviate from s^* to induce a
  - but then s^* is a NE at θ_2
- Indeed f is not strong EM!

Universal Distinguishability

Back

Proof.

- $\forall\theta$, we build $e^*_\theta \in E^\theta$ so that any f is evidence-monotonic

  i.e. such that for any $\theta' \neq \theta$

$$e^*_\theta \notin E^{\theta'} \text{ or } E^{\theta'} \not\subseteq E^\theta$$

- Fix θ, and let $\Omega = \Theta \setminus \{\theta\}$. By Univ. Distinguishability,

$$\bigcup_{\theta' \in \Omega} E^{\theta'} \neq E^\theta$$

- Case 1: "$\not\supseteq$" $\implies$ pick $e^*_\theta \notin E^{\theta'}\ \forall\theta' \neq \theta \implies$ we are done
- Case 2: "$\not\subseteq$" $\implies \exists\theta' \neq \theta : E^{\theta'} \not\subseteq E^\theta \implies$ knock out θ'
- Let $\Omega = \Theta \setminus \{\theta, \theta'\}$; iterate the reasoning $\cdots$ $\square$

Costly Evidence: the Mechanism

Back

Build on the mechanism used earlier. For each i : $M_i = \Theta \times A \times \mathbb{N}$.

- **Rule 1:** If $m_1 = \cdots = m_n = (\theta, f(\theta), k)$ and $e = e^*_\theta$ $\implies$ outcome is $f(\theta)$ and no transfers
- **Rule 2:** If for some i: $m_j = (\theta, f(\theta), k)$ and $e_j = e^*_{j,\theta}$ for all $j \neq i$ while $(m_i, e_i) = (\theta_i, b_i, k_i, e_i) \neq (\theta, f(\theta), k, e^*_{i,\theta})$
  - Case (a): $e_i \in E_i^\theta \implies$ worst outcome for i under state θ between b_i and $f(\theta)$, and no transfers
  - Case (b): $e_i \notin E_i^\theta \implies$ pick $f(\theta)$ and reward i with transfer $= c_i(e_i, \theta)$ [can also balance budget]
- **Rule 3:** For any other case, no transfers and choose outcome announced by player with highest integer

Weak Non-satiation

◂ NS

For each ordered pair of states (θ, θ'), let

$$D\left(\theta, \theta'\right) := \left\{ i \in I : E_i^{\theta'} \not\subseteq E_i^{\theta} \right\}.$$

SCF f satisfies weak non-satiation if $\forall \theta, \theta'$ s.t. $D(\theta, \theta') \neq \emptyset$,

$$\exists i \in D\left(\theta, \theta'\right) \text{ and } a \in A \text{ s.t. } u_i(a, \theta') > u_i(f(\theta), \theta').$$

Dynamic Mechanisms

Bull and Watson (2007):

Under the feasibility interpretation of hard evidence, dynamic mechanisms can be helpful for (Nash-)implementation

- 2 players; three states, $\Theta = \{\theta_1, \theta_2, \theta_3\}$
- State independent preferences: $b \succ a$ by 1; $a \succ b$ by 2
- Evidence structure:
  - Player 1: $E_1^{\theta_1} = E_1^{\theta_2} = E_1^{\theta_3}$
  - Player 2: $E_2^{\theta_1} = \{x\}; E_2^{\theta_2} = \{x, y\}; E_2^{\theta_3} = \{y\}$
- $f(\theta_1) = f(\theta_3) = b$ and $f(\theta_2) = a$

This SCF *is not* evidence-monotonic:

$$T^f(\theta_2) = \{\theta_1, \theta_3\} \text{ but } \theta_2 \text{ is not distinguishable from } \{\theta_1, \theta_3\}$$

Dynamic Mechanisms

$b \succ a$ by 1; $a \succ b$ by 2

$E_2^{\theta_1} = \{x\}; E_2^{\theta_2} = \{x, y\}; E_2^{\theta_3} = \{y\}$

$f(\theta_1) = f(\theta_3) = b$ and $f(\theta_2) = a$

Under feasibility of hard evidence interpretation, f is implemented (in NE) by the following dynamic mechanism:

1st Stage: player 1 can announce any state $\theta \in \Theta$.

2nd Stage: after observing player 1's announcement, player 2 has to send evidence.

Outcomes: $g(\theta_2, e) = a$ for any $e \in \{x, y\}$, and if $\theta \neq \theta_2$:

$$g(\theta, e) = \begin{cases} b \text{ for } e \in E_2^{\theta} \\ a \text{ otherwise.} \end{cases}$$

But this doesn't work under the cost interpretation.

Dynamic Mechanisms

Proposition.

Dynamic mechanisms are not helpful for Nash-implementation if either

1. *the evidence structure is normal,*

   or

2. $\forall i, \theta$: $e_i \notin E_i^\theta$ *can be produced at* θ *but is infinitely costly.*

Intuition.

Dynamic mechanisms not helpful because:

1. *Under normality, no need to have players tailor their evidence submission to what others have submitted*

2. *Under cost interpretation, incredible threats can be used (since these not ruled out by Nash equilibrium)*