Motivation

Primary instrument to control elected officials: replacement

Replacement serves two roles:

1. Provide incentives
2. Selection

Other settings: organizational leaders, bureaucrats, doctors, etc.

Motivation

Primary instrument to control elected officials: replacement

Replacement serves two roles:

1. Provide incentives
2. Selection

Other settings: organizational leaders, bureaucrats, doctors, etc.

Our question: Does replacement lead to good outcomes in the long run?

This paper

A stylized model of accountability with moral hazard and adverse selection

- Reputational model: long-lived politicians; short-lived voters
- Replacement is the only source of incentives

This paper

A stylized model of accountability with moral hazard and adverse selection

- Reputational model: long-lived politicians; short-lived voters
- Replacement is the only source of incentives

Results

- There is always an eqm with good outcomes in the long run
- Under some conditions, there is an eqm with good outcomes in every period

This paper

A stylized model of accountability with moral hazard and adverse selection

- Reputational model: long-lived politicians; short-lived voters
- Replacement is the only source of incentives

Results

- There is always an eqm with good outcomes in the long run
- Under some conditions, there is an eqm with good outcomes in every period
- But those same conditions also characterize when
  there are eqa in which good outcomes do <u>not</u> obtain even in the long run

This paper

A stylized model of accountability with moral hazard and adverse selection

- Reputational model: long-lived politicians; short-lived voters
- Replacement is the only source of incentives

Results

- There is always an eqm with good outcomes in the long run
- Under some conditions, there is an eqm with good outcomes in every period
- But those same conditions also characterize when
  there are eqa in which good outcomes do not obtain even in the long run

Takeaways

- Replacement makes long-run good outcomes possible (in some eqm) and,
  sometimes guarantees it (in all eqa) $\rightarrow$ guarantee operates through selection
- Tension between that guarantee and the possibility of good outcomes in all periods

This paper

A stylized model of accountability with moral hazard and adverse selection

- Reputational model: long-lived politicians; short-lived voters
- Replacement is the only source of incentives

Results

- There is always an eqm with good outcomes in the long run
- Under some conditions, there is an eqm with good outcomes in every period
- But those same conditions also characterize when
  there are eqa in which good outcomes do not obtain even in the long run

Takeaways

- Replacement makes long-run good outcomes possible (in some eqm) and,
  sometimes guarantees it (in all eqa) $\rightarrow$ guarantee operates through selection
- Tension between that guarantee and the possibility of good outcomes in all periods
- Source of long-run inefficiencies is “excessive” replacement

Related Literature

Political accountability with selection

- Banks & Sundaram (1993); Fearon (1999); Myerson (2006); Anesi & Buisseret (2022)

Reputation with imperfect monitoring

- Fudenberg & Levine (1992); Cripps, Mailath, Samuelson (2004)

Related Literature

Political accountability with selection

- Banks & Sundaram (1993); Fearon (1999); Myerson (2006); Anesi & Buisseret (2022)

Reputation with imperfect monitoring

- Fudenberg & Levine (1992); Cripps, Mailath, Samuelson (2004)

Reputation with exogenous replacement

- Mailath & Samuelson (2001); Tadelis (2002); Ekmekci, Gossner, Wilson (2012)

Reputation with competition between long-lived players

- Hörner (2002); Atakan & Ekmekci (2015); Deb & Fanning (2024)

Model

Model

Time: $t = 0, 1, \ldots$

Pool of infinitely many identical long-lived politicians

Sequence of short-lived voters, one in each period

Model

Time: $t = 0, 1, \ldots$

Pool of infinitely many identical long-lived politicians

Sequence of short-lived voters, one in each period

In period 0, one politician is exogenously the incumbent

Incumbent chooses effort $a_0 \in \{0, 1\}$ and generates public signal $s_0 \sim f(\cdot|a_0) \in \Delta S$,

- S is finite, $f(\cdot|0) \neq f(\cdot|a = 1)$, and $f(\cdot|1)$ has full support

Model

Time: $t = 0, 1, \ldots$

Pool of infinitely many identical long-lived politicians

Sequence of short-lived voters, one in each period

In period 0, one politician is exogenously the incumbent

Incumbent chooses effort $a_0 \in \{0, 1\}$ and generates public signal $s_0 \sim f(\cdot|a_0) \in \Delta S$,

- S is finite, $f(\cdot|0) \neq f(\cdot|a = 1)$, and $f(\cdot|1)$ has full support

Period $t \geq 1$:

- Current voter decides whether to replace the incumbent with random new draw from pool
- Incumbent (old or new) chooses $a_t \in \{0, 1\}$ and generates public signal $s_t \sim f(\cdot|a_t)$

Model

Time: $t = 0, 1, \dots$

Pool of infinitely many identical long-lived politicians

Sequence of short-lived voters, one in each period

In period 0, one politician is exogenously the incumbent

Incumbent chooses effort $a_0 \in \{0, 1\}$ and generates public signal $s_0 \sim f(\cdot|a_0) \in \Delta S$,

- S is finite, $f(\cdot|0) \neq f(\cdot|a=1)$, and $f(\cdot|1)$ has full support

Period $t \geq 1$:

- Current voter decides whether to replace the incumbent with random new draw from pool
- Incumbent (old or new) chooses $a_t \in \{0, 1\}$ and generates public signal $s_t \sim f(\cdot|a_t)$

Replacements are public; once a politician is replaced, he never returns

Payoffs and Types

Voter t's payoff is a_t — just wants incumbent effort

- Paper also allows for replacement cost (not too large)

Payoffs and Types

Voter t's payoff is a_t — just wants incumbent effort

- Paper also allows for replacement cost (not too large)

Each politician is either good/committed or opportunistic/normal/rational

Politicians' types are private info and independent; each is good with prob $\pi_0 \in (0, 1)$

Payoffs and Types

Voter t's payoff is a_t — just wants incumbent effort

- Paper also allows for replacement cost (not too large)

Each politician is either good/committed or opportunistic/normal/rational

Politicians' types are private info and independent; each is good with prob $\pi_0 \in (0, 1)$

- Good type exerts effort (action 1) whenever he is in office

Payoffs and Types

Voter t's payoff is a_t — just wants incumbent effort

- Paper also allows for replacement cost (not too large)

Each politician is either good/committed or opportunistic/normal/rational

Politicians' types are private info and independent; each is good with prob $\pi_0 \in (0,1)$

- Good type exerts effort (action 1) whenever he is in office
- Opportunistic type's stage payoff is

$$u_s = \begin{cases} 0 & \text{if not in office in period } s \\ u(a_s) & \text{if in office in period } s, \end{cases}$$

where $u(0) > u(1) > 0$.

His overall payoff is $(1-\delta)\sum_{s=0}^{\infty} \delta^s u_s$, with $\delta \in (0,1)$

Strategies & Solution Concept

Equilibrium: symmetric weak PBE, i.e., weak PBE in which

- All opportunistic politicians use the same strategy:
  sequence of own signals $\rightarrow$ prob of effort
- All voters use same strategy:
  sequence of incumbent's own signals $\rightarrow$ prob of replacement

Strategies & Solution Concept

Equilibrium: symmetric weak PBE, i.e., weak PBE in which

- All opportunistic politicians use the same strategy:
  sequence of own signals $\rightarrow$ prob of effort
- All voters use same strategy:
  sequence of incumbent's own signals $\rightarrow$ prob of replacement

What are we ruling out?

- Coordination on previous office-holders' signals / when an incumbent took office
- Different politicians using different strategies

$\rightarrow$ *role of replacement is to start interaction with new (ex-ante identical) office-holder afresh*

Strategies & Solution Concept

Equilibrium: symmetric weak PBE, i.e., weak PBE in which

- All opportunistic politicians use the same strategy:
  sequence of own signals $\rightarrow$ prob of effort
- All voters use same strategy:
  sequence of incumbent's own signals $\rightarrow$ prob of replacement

What are we ruling out?

- Coordination on previous office-holders' signals / when an incumbent took office
- Different politicians using different strategies

$\rightarrow$ *role of replacement is to start interaction with new (ex-ante identical) office-holder afresh*

Why weak PBE, rather than PBE?

- Strengthens one direction of main result
  (property of all equilibria)
- But off-path flexibility plays role in other direction
  (construction of a bad equilibrium)

Strategies & Solution Concept

Equilibrium: symmetric weak PBE, i.e., weak PBE in which

- All opportunistic politicians use the same strategy:
  sequence of own signals $\rightarrow$ prob of effort
- All voters use same strategy:
  sequence of incumbent's own signals $\rightarrow$ prob of replacement

What are we ruling out?

- Coordination on previous office-holders' signals / when an incumbent took office
- Different politicians using different strategies

$\rightarrow$ *role of replacement is to start interaction with new (ex-ante identical) office-holder afresh*

Why weak PBE, rather than PBE?

- Strengthens one direction of main result
  (property of all equilibria)
- But off-path flexibility plays role in other direction
  (construction of a bad equilibrium)

Proposition

A symmetric PBE exists.

Idea: auxiliary game with just one politician; a voter's payoff on replacement is $\pi_0 + (1 - \pi_0)\sigma_P(\emptyset)$

Main Result

(Eventual) First Best

Definition

An equilibrium attains first best if $\mathbb{P}(a_t = 1) = 1$ for all $t \geq 0$.

(Eventual) First Best

Definition

An equilibrium attains first best if $\mathbb{P}(a_t = 1) = 1$ for all $t \geq 0$.

Definition

An equilibrium attains eventual first best if a.s. there is some τ s.t. $a_t = 1$ for all $t \geq \tau$.

Main Result: An Equivalence Theorem

If an eqm attains FB, it has no learning; so FB must also be attainable absent good type

Intuitively, this requires conjunction of

low-enough effort cost; sufficiently-informative monitoring; enough patience

Formally on next slide $\rightarrow$ Condition FB-I

Main Result: An Equivalence Theorem

If an eqm attains FB, it has no learning; so FB must also be attainable absent good type

Intuitively, this requires conjunction of

low-enough effort cost; sufficiently-informative monitoring; enough patience

Formally on next slide → Condition FB-I

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB

Main Result: An Equivalence Theorem

If an eqm attains FB, it has no learning; so FB must also be attainable absent good type

Intuitively, this requires conjunction of

low-enough effort cost; sufficiently-informative monitoring; enough patience

Formally on next slide $\rightarrow$ Condition FB-I

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB
3. All equilibria attain Eventual FB
4. $\mathbb{P}(a_0 = 1) < 1$ in all equilibria

Main Result: An Equivalence Theorem

If an eqm attains FB, it has no learning; so FB must also be attainable absent good type

Intuitively, this requires conjunction of

low-enough effort cost; sufficiently-informative monitoring; enough patience

Formally on next slide → Condition FB-I

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB
3. All equilibria attain Eventual FB
4. $\mathbb{P}(a_0 = 1) < 1$ in all equilibria

So, always some eqm that attains Eventual FB

But there is a tension between:

- Eventual FB in all eqa
- FB (or even FB in first period) in some eqm

Condition FB-I

Condition FB-I

There is a vector $(v(s))_{s\in S} \in [0, u(0)]^S$ such that

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq (1-\delta)u(0) + \delta \sum_{s\in S} f(s|0)v(s) \tag{IC$_{\mathsf{FB}}$}$$

Condition FB-I

Condition FB-I

There is a vector $(v(s))_{s\in S} \in [0, u(0)]^S$ such that

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq (1-\delta)u(0) + \delta \sum_{s\in S} f(s|0)v(s) \tag{IC$_{\mathsf{FB}}$}$$

and

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq \max_{s\in S} v(s). \tag{PK$_{\mathsf{FB}}$}$$

Condition FB-I

Condition FB-I

There is a vector $(v(s))_{s\in S} \in [0, u(0)]^S$ such that

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq (1-\delta)u(0) + \delta \sum_{s\in S} f(s|0)v(s) \qquad (\mathrm{IC_{FB}})$$

and

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq \max_{s\in S} v(s). \qquad (\mathrm{PK_{FB}})$$

- Evidently necessary for a FB eqm
- Also sufficient

Condition FB-I

Condition FB-I

There is a vector $(v(s))_{s\in S} \in [0, u(0)]^S$ such that

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq (1-\delta)u(0) + \delta \sum_{s\in S} f(s|0)v(s) \tag{IC$_{\text{FB}}$}$$

and

$$(1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) \geq \max_{s\in S} v(s). \tag{PK$_{\text{FB}}$}$$

- Evidently necessary for a FB eqm
- Also sufficient

Holds if and only if

- $u(0) - u(1)$ is small enough; and
- f is sufficiently informative; and
- δ is large enough

First-Period Outcomes when Condition FB-I Fails

Theorem says that $\mathbb{P}(a_0 = 1) < 1$ in all equilibria when Condition FB-I fails

How bad can period 0 be for voters? (Same as first period for every new incumbent.)

First-Period Outcomes when Condition FB-I Fails

Theorem says that $\mathbb{P}(a_0 = 1) < 1$ in all equilibria when Condition FB-I fails

How bad can period 0 be for voters? (Same as first period for every new incumbent.)

Proposition

Suppose $\{u, \delta, f\}$ violates Condition FB-I.

For every $\varepsilon > 0$, there exists $\overline{\pi}_0 \in (0, 1)$ such that if $\pi_0 < \overline{\pi}_0$, then $\mathbb{P}(a_0 = 1) < \varepsilon$ in all equilibria.

I.e., absent Condition FB-I and if good types are unlikely, then negligible effort from any newly-installed incumbent

What Prevents Good Long-Run Outcomes?

Theorem says some eqa do not attain Eventual FB even when Condition FB-I holds.

What causes the failure of Eventual FB in those equilibria?

What Prevents Good Long-Run Outcomes?

Theorem says some eqa do not attain Eventual FB even when Condition FB-I holds.

What causes the failure of Eventual FB in those equilibria?

Let $\pi(h^t)$ denote the incumbent's reputation at history h^t. (Probability of good type.)

Proposition

Consider any equilibrium that does not attain Eventual FB.

There is a positive-prob history h^t at which
$\pi(h^t) > \pi_0$ and yet the incumbent is replaced with positive prob.

In this sense, Eventual FB can only be prevented by "too much" replacement

What Prevents Good Long-Run Outcomes?

Theorem says some eqa do not attain Eventual FB even when Condition FB-I holds.

What causes the failure of Eventual FB in those equilibria?

Let $\pi(h^t)$ denote the incumbent's reputation at history h^t. (Probability of good type.)

Proposition

Consider any equilibrium that does not attain Eventual FB.

There is a positive-prob history h^t at which
$\pi(h^t) > \pi_0$ and yet the incumbent is replaced with positive prob.

In this sense, Eventual FB can only be prevented by "too much" replacement

Contrast with the reason for failure of Eventual FB in Myerson (2006)

- There, replacement cost is too high, so there is not enough replacement

What Assures Good Long-Run Outcomes?

Theorem says all eqa attain Eventual FB when Condition FB-I fails.

It turns out this obtains via asymptotic selection.

What Assures Good Long-Run Outcomes?

Theorem says all eqa attain Eventual FB when Condition FB-I fails.

It turns out this obtains via asymptotic selection.

Let π_t denote probability that period-t incumbent is a good type.

Proposition

In every eqm:

1. Each opportunistic office-holder is replaced a.s. as $t \to \infty$.
2. If Condition FB-I fails, then each good-type office-holder is retained forever with pos prob, and moreover, $\lim_{t\to\infty} \pi_t = 1$ a.s.

Logic for the first part is closely related to CMS (2004), "impermanent reputations"

Proofs Ideas

Proof Plan

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB
3. All equilibria attain Eventual FB
4. $\mathbb{P}(a_0 = 1) < 1$ in all equilibria

Proof Plan

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB
3. All equilibria attain Eventual FB
4. $\mathbb{P}(a_0 = 1) < 1$ in all equilibria

I will talk about:

- not (1) $\implies$ not (2) [hence also not (4)]
- not (1) $\implies$ not (3)
- (1) $\implies$ (3)

Proof Plan

Theorem

The following are equivalent:

1. $\{u, f, \delta\}$ violates Condition FB-I
2. No equilibrium attains FB
3. All equilibria attain Eventual FB
4. $\mathbb{P}(a_0 = 1) < 1$ in all equilibria

I will talk about:

- not (1) $\implies$ not (2) [hence also not (4)]
- not (1) $\implies$ not (3)
- (1) $\implies$ (3)

Omit:

- (1) $\implies$ (2) is clear
- (1) $\implies$ (4)

Condition FB-I $\implies$ Equilibria that Attain FB

Consider the following strategy profile

- Incumbent always exerts effort on the eqm path.
- Voter t replaces the incumbent if and only signal s_{t-1} has a LR below some threshold.
- Off path, voter never retains incumbent and incumbent always shirks.

Condition FB-I $\implies$ Equilibria that Attain FB

Consider the following strategy profile

- Incumbent always exerts effort on the eqm path.
- Voter t replaces the incumbent if and only signal s_{t-1} has a LR below some threshold.
- Off path, voter never retains incumbent and incumbent always shirks.

We show that Condition FB-I $\implies$ there is a threshold s.t. this profile is an eqm.

▸ Formal

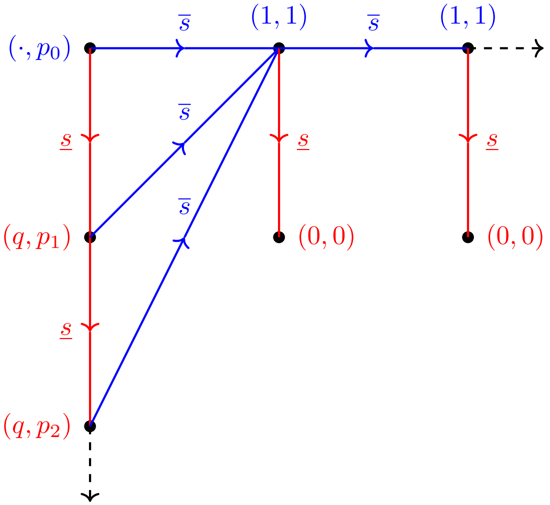
- Only thing to check is incumbent's incentive

Condition FB-I $\implies$ Equilibria that Fail Eventual FB

Construction with two signals, $S = \{\underline{s}, \overline{s}\}$.

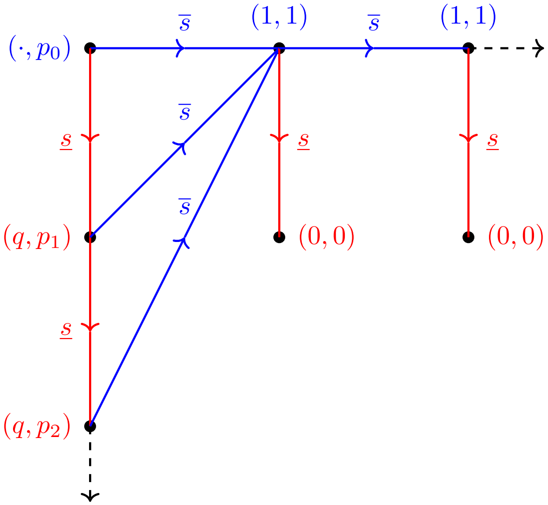
Condition FB-I $\implies$ Equilibria that Fail Eventual FB

Construction with two signals, $S = \{\underline{s}, \overline{s}\}$. The first number in the vector is the prob of retention, and the second number is the prob of choosing $a = 1$. It holds that $0 < p_0 < p_1 < \cdots < 1$ and $q \in [0, 1)$.



Condition FB-I $\Longrightarrow$ Equilibria that Fail Eventual FB

Construction with two signals, $S = \{\underline{s}, \overline{s}\}$. The first number in the vector is the prob of retention, and the second number is the prob of choosing $a = 1$. It holds that $0 < p_0 < p_1 < \cdots < 1$ and $q \in [0, 1)$.



Caveat:
use weak PBE to sustain $(0,0)$ profiles

Condition FB-I Fails $\implies$ Eventual FB in All Equilibria

Two Steps:

Condition FB-I Fails $\implies$ Eventual FB in All Equilibria

Two Steps:

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Condition FB-I Fails $\implies$ Eventual FB in All Equilibria

Two Steps:

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,
with pos prob the period-0 incumbent is never replaced.

Condition FB-I Fails $\implies$ Eventual FB in All Equilibria

Two Steps:

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,
with pos prob the period-0 incumbent is never replaced.

Step 2 $\implies$ a.s., some politician will stay in office forever

$\because$ all period-0 incumbents have same prob of never being replaced (symmetric eqm)

Step 1 $\implies$ it is a good type, and hence Eventual FB

Either Replaced or Action Converges to 1

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,

with probability 1 any opportunistic incumbent is replaced.

Either Replaced or Action Converges to 1

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,

with probability 1 any opportunistic incumbent is replaced.

Let π_t denote an incumbent's reputation in period t.

- If he is replaced at $s < t$, then let $\pi_t \equiv \pi_s$

Since $\{\pi_t\}_{t \geq 0}$ is a bounded martingale, it converges to some π_∞ a.s.

Either Replaced or Action Converges to 1

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Let π_t denote an incumbent's reputation in period t.

- If he is replaced at $s < t$, then let $\pi_t \equiv \pi_s$

Since $\{\pi_t\}_{t \geq 0}$ is a bounded martingale, it converges to some π_∞ a.s.

Case 1: $\pi_\infty > 0$

Then $a_t \to 1$ (conditional on not being replaced)
otherwise revealed as opportunistic, contradicting $\pi_\infty > 0$

But such effort from opportunistic type requires (time-averaged) replacement hazard rate bounded away from zero

Hence, by Borel-Cantelli, the opportunistic type is replaced with probability 1

Either Replaced or Action Converges to 1

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Case 2: $\pi_\infty = 0$

Lemma

Suppose Condition FB-I fails. In every eqm,
there exists $\eta > 0$ such that the incumbent will be replaced for sure once $\pi_t < \eta$.

Either Replaced or Action Converges to 1

Lemma (Step 1)

Suppose Condition FB-I fails. In every eqm,
with probability 1 any opportunistic incumbent is replaced.

Case 2: $\pi_\infty = 0$

Lemma

Suppose Condition FB-I fails. In every eqm,
there exists $\eta > 0$ such that the incumbent will be replaced for sure once $\pi_t < \eta$.

Suppose not.

Voter's willingness not to replace incumbent when $\pi(h) \approx 0$
$\implies$ opportunistic type must exert effort with pos prob

Condition FB-I fails $\implies$ $\exists \varepsilon > 0$ and $s_1 \in S$ s.t. $V(h, s_1) \geq V(h) + \varepsilon > 0$

Iterating this logic some T times, noting that $\pi(h, s_1, \ldots, s_T) \approx 0$, contradicts $V(\cdot) \leq u(0)$

Towards Proving Never Replaced With Pos Prob

Let $\mathcal{H}_*$ denote the set of histories s.t.

- The first incumbent reaches that history with positive prob
- The incumbent is retained at that history with positive prob

Let $\overline{S} \equiv \{s \in S | f(s|1) > f(s|0)\}$, i.e., the set of good signals.

Lemma

For every $h \in \mathcal{H}_*$, there exists $s \in \overline{S}$ such that $(h, s) \in \mathcal{H}_*$.

Towards Proving Never Replaced With Pos Prob

Let $\mathcal{H}_*$ denote the set of histories s.t.

- The first incumbent reaches that history with positive prob
- The incumbent is retained at that history with positive prob

Let $\overline{S} \equiv \{s \in S | f(s|1) > f(s|0)\}$, i.e., the set of good signals.

Lemma

For every $h \in \mathcal{H}_*$, there exists $s \in \overline{S}$ such that $(h, s) \in \mathcal{H}_*$.

Consider two cases:

Towards Proving Never Replaced With Pos Prob

Let $\mathcal{H}_*$ denote the set of histories s.t.

- The first incumbent reaches that history with positive prob
- The incumbent is retained at that history with positive prob

Let $\overline{S} \equiv \{s \in S | f(s|1) > f(s|0)\}$, i.e., the set of good signals.

Lemma

For every $h \in \mathcal{H}_*$, there exists $s \in \overline{S}$ such that $(h, s) \in \mathcal{H}_*$.

Consider two cases:

1. Opportunistic type does not exert effort at h.

   After observing $s \in \overline{S}$, we have $\pi(h, s) > \pi(h)$.

   Since incumbent was retained with pos prob at h, it follows that $(h, s) \in \mathcal{H}_*$.

Towards Proving Never Replaced With Pos Prob

Let $\mathcal{H}_*$ denote the set of histories s.t.

- The first incumbent reaches that history with positive prob
- The incumbent is retained at that history with positive prob

Let $\overline{S} \equiv \{s \in S | f(s|1) > f(s|0)\}$, i.e., the set of good signals.

Lemma

For every $h \in \mathcal{H}_*$, there exists $s \in \overline{S}$ such that $(h, s) \in \mathcal{H}_*$.

Consider two cases:

2. Opportunistic type exerts effort with pos prob at h.

   Since effort is costly and increases prob of good signals, $V(h, s) > 0$ for some $s \in \overline{S}$, and so $(h, s) \in \mathcal{H}_*$.

Towards Proving Never Replaced With Pos Prob

Let $\overline{\pi} \equiv \sup_{h \in \mathcal{H}_*} \pi(h)$.

Lemma

If Condition FB-I fails, then $\overline{\pi} = 1$ in all equilibria.

Towards Proving Never Replaced With Pos Prob

Let $\overline{\pi} \equiv \sup_{h \in \mathcal{H}_*} \pi(h)$.

Lemma

If Condition FB-I fails, then $\overline{\pi} = 1$ in all equilibria.

Suppose by way of contradiction that $\overline{\pi} < 1$. By definition of $\overline{\pi}$,

- For every $\eta > 0$, there exists $h \in \mathcal{H}_*$ s.t. $\pi(h) > \overline{\pi} - \eta$.

Towards Proving Never Replaced With Pos Prob

Let $\overline{\pi} \equiv \sup_{h \in \mathcal{H}_*} \pi(h)$.

Lemma

If Condition FB-I fails, then $\overline{\pi} = 1$ in all equilibria.

Suppose by way of contradiction that $\overline{\pi} < 1$. By definition of $\overline{\pi}$,

- For every $\eta > 0$, there exists $h \in \mathcal{H}_*$ s.t. $\pi(h) > \overline{\pi} - \eta$.

Fix a small enough $\eta > 0$.

At h, the opportunistic type needs to exert effort with high probability.

Towards Proving Never Replaced With Pos Prob

Let $\overline{\pi} \equiv \sup_{h \in \mathcal{H}_*} \pi(h)$.

Lemma

If Condition FB-I fails, then $\overline{\pi} = 1$ in all equilibria.

Suppose by way of contradiction that $\overline{\pi} < 1$. By definition of $\overline{\pi}$,

- For every $\eta > 0$, there exists $h \in \mathcal{H}_*$ s.t. $\pi(h) > \overline{\pi} - \eta$.

Fix a small enough $\eta > 0$.

At h, the opportunistic type needs to exert effort with high probability. Why?

- There exists $\overline{s} \in \overline{S}$ such that $(h, \overline{s}) \in \mathcal{H}_*$.
- If the opportunistic type does not exert effort with high prob, then $\pi(h, \overline{s}) > \overline{\pi}$, a contradiction.

Hence, $\pi(h, s) \approx \pi(h)$ for all $s \in S$

Towards Proving Never Replaced With Pos Prob

Let $\overline{\pi} \equiv \sup_{h \in \mathcal{H}_*} \pi(h)$.

Lemma

If Condition FB-I fails, then $\overline{\pi} = 1$ in all equilibria.

Suppose by way of contradiction that $\overline{\pi} < 1$. By definition of $\overline{\pi}$,

- For every $\eta > 0$, there exists $h \in \mathcal{H}_*$ s.t. $\pi(h) > \overline{\pi} - \eta$.

Fix a small enough $\eta > 0$.

At h, the opportunistic type needs to exert effort with high probability. Why?

- There exists $\overline{s} \in \overline{S}$ such that $(h, \overline{s}) \in \mathcal{H}_*$.
- If the opportunistic type does not exert effort with high prob, then $\pi(h, \overline{s}) > \overline{\pi}$, a contradiction.

Hence, $\pi(h, s) \approx \pi(h)$ for all $s \in S$

Condition FB-I fails $\implies$ $\exists \varepsilon > 0$ and $s_1 \in S$ such that $V(h, s_1) \geq V(h) + \varepsilon$

Iterating same logic at (h, s_1) and onward, reach a contradiction since $V(\cdot) \leq u(0)$.

Never Replaced With Pos Prob

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,
with pos prob the period-0 incumbent is never replaced.

When Condition FB-I fails, $\mathbb{P}(a_0 = 1) < 1$ in all equilibria.

- Another iteration of increasing continuation values argument

▸ Details

Never Replaced With Pos Prob

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,
with pos prob the period-0 incumbent is never replaced.

When Condition FB-I fails, $\mathbb{P}(a_0 = 1) < 1$ in all equilibria.

- Another iteration of increasing continuation values argument

▸ Details

Fix any equilibrium. $\exists \eta > 0$ s.t. voters will not replace incumbent at h if $\pi(h) > 1 - \eta$.

Never Replaced With Pos Prob

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,

with pos prob the period-0 incumbent is never replaced.

When Condition FB-I fails, $\mathbb{P}(a_0 = 1) < 1$ in all equilibria.

- Another iteration of increasing continuation values argument

Details

Fix any equilibrium. $\exists \eta > 0$ s.t. voters will not replace incumbent at h if $\pi(h) > 1 - \eta$.

Since $\overline{\pi} = 1$, π_t reaches $1 - \eta/2$ with positive probability.

Never Replaced With Pos Prob

Lemma (Step 2)

Suppose Condition FB-I fails. In every eqm,
with pos prob the period-0 incumbent is never replaced.

When Condition FB-I fails, $\mathbb{P}(a_0 = 1) < 1$ in all equilibria.

- Another iteration of increasing continuation values argument

▸ Details

Fix any equilibrium. $\exists \eta > 0$ s.t. voters will not replace incumbent at h if $\pi(h) > 1 - \eta$.

Since $\overline{\pi} = 1$, π_t reaches $1 - \eta/2$ with positive probability.

Doob's upcrossing inequality implies that
conditional on $\pi_t \geq 1 - \eta/2$, event $\{\pi_s \geq 1 - \eta \text{ for every } s \geq t\}$ occurs with pos prob.

Conclusion

Conclusion

A model of accountability with moral hazard and adverse selection

- Each politician is either good or opportunistic
- Replacement is an instrument to provide incentives and to select good politicians

Takeaways

- Replacement makes long-run good outcomes possible (in some eqm) and, sometimes guarantees it (in all eqa)
- Tension between that guarantee and the possibility of good outcomes in all periods
- Source of long-run inefficiencies is "excessive" replacement

Appendix

Condition FB-I Fails $\implies \mathbb{P}(a_0 = 1) < 1$ in every eqm

If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$ (Back)

Condition FB-I Fails $\implies \mathbb{P}(a_0 = 1) < 1$ in every eqm

If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$ (Back)

Consider any $\varepsilon > 0$ small enough.

Because Condition FB-I fails, there exists $s_0 \in S$ such that $V(s_0) \geq V(\emptyset) + \varepsilon$.

Condition FB-I Fails $\implies \mathbb{P}(a_0 = 1) < 1$ in every eqm

If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$ (Back)

Consider any $\varepsilon > 0$ small enough.

Because Condition FB-I fails, there exists $s_0 \in S$ such that $V(s_0) \geq V(\emptyset) + \varepsilon$.

Hence, at history s_0, the incumbent is retained with pos prob

- Voter optimality $\implies$ the opportunistic type must again be incentivized to play $a_1 = 1$
- There exists $s_1 \in S$ such that $V(s_0, s_1) \geq V(s_0) + \varepsilon$.

Condition FB-I Fails $\implies \mathbb{P}(a_0 = 1) < 1$ in every eqm

If $\mathbb{P}(a_0 = 1) = 1$, then the opportunistic type must be incentivized to play $a_0 = 1$ (Back)

Consider any $\varepsilon > 0$ small enough.

Because Condition FB-I fails, there exists $s_0 \in S$ such that $V(s_0) \geq V(\emptyset) + \varepsilon$.

Hence, at history s_0, the incumbent is retained with pos prob

- Voter optimality $\implies$ the opportunistic type must again be incentivized to play $a_1 = 1$
- There exists $s_1 \in S$ such that $V(s_0, s_1) \geq V(s_0) + \varepsilon$.

Iterating, for any $t \geq 0$, there is a pos-prob history $(s_0, \ldots, s_t)$ s.t. $V(s_0, \ldots, s_t) \geq V(\emptyset) + t\varepsilon$

Contradiction, as $V(\cdot) \leq u(0)$

Details for Condition FB-I $\implies$ Eqm that Attains FB

Lemma

Assume Condition FB-I. There exist $\overline{v} > 0$ and $x \in (0, 1]$ such that (IC_{FB}) holds and

$$(1-\delta)u(1) + \delta \sum_{s \in S} f(s|1)v(s) = \overline{v},$$

with

$$v(s) = \begin{cases} \overline{v} & \text{if} \quad \frac{f(s|1)}{f(s|0)} \geq x \\ 0 & \text{if} \quad \frac{f(s|1)}{f(s|0)} < x. \end{cases}$$

(Back)

Incentive Lemma when Condition FB-I Fails

Lemma

If Condition FB-I fails, then $\exists \varepsilon > 0$ s.t. $\forall \left(v(s)\right)_{s\in S} \in [0, u(0)]^S$ that satisfies (IC_{FB}),

$$\max_{s\in S} v(s) \geq (1-\delta)u(1) + \delta \sum_{s\in S} f(s|1)v(s) + \varepsilon.$$