Information Aggregation and Sequential Voting

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Theory ("Social Learning in Elections")
with Nageeb Ali

&

Experiments (work in progress)
with Nageeb Ali, Jacob Goeree, and Thomas Palfrey

Introduction

- Most models of voting are static
- But many elections feature dynamic/sequential voting
- Roll-call voting in city councils, legislatures, boards
- U.S. Presidential Primaries
- EU Constitution Referendum, 2005
  - France (May) and Netherlands (June) voted “No”
  - Many argued that this would have a domino effect

Introduction

- Generally, momentum effects in sequential elections
- We develop an informational theory of rational momentum and observational learning in elections
- Why study an information-based explanation?
  - Elections as information aggregation mechanisms
  - Matters in practice (Knight and Schiff, 2007)

    "When New Yorkers go to vote next Tuesday, they cannot help but be influenced by Kerry's victories in Wisconsin last week. **Surely those Wisconsinites knew something, and if so many of them voted for Kerry, then he must be a decent candidate**."

    – Duncan Watts in *Slate* Magazine (2004)

Don't We Already Know the Answer?

Standard Cascades Model	Sequential Voting
Only individual action matters	*Collective* action matters
Only externality is informational	*Both* informational and action externalities
No forward-looking incentive	*Yes*, forward-looking incentive
⇓	⇓
Herding is uniquely rational	??

Our Contribution

We show that in a canonical sequential voting model, there is a history-dependent equilibrium that generates bandwagons with probability 1 in large elections.

- Voter behavior takes a simple form
- Provides rational underpinnings for momentum effects
- Interesting comparative statics & other implications
- No counterpart in simultaneous elections

Related Literature

	No payoff interdependencies	Voting
Sim. actions		ASB (1996) FP (1996, 1997) Myerson (1998)
Seq. actions	Banerjee (1992) BHW (1992) SS (2000)	Dekel-Piccione (2000) Callander (2007) Fey (2000), Wit (1997)

Related Literature

- Fey (2000) and Wit (1997): first to study such models

- Dekel-Piccione (2000) show that history *independent* equilibria exist in symmetric sequential voting games

  ... but easy to misinterpret as full-equivalence result.

- Callander (2007) derives bandwagon equilibria assuming
  - an infinite number of voters
  - a preference for conformity

Plan

Model

Posterior-Based Voting

Implications

Experiments

Model: Basics

- Two exogenous candidates, L and R
- Voters $i = 1, \ldots, n$ (n odd)
- Vote in roll-call sequence, observing history, no abstention
- Majority rule $\implies$ winner, W
- Two states, $\omega \in \{L, R\}$, prior $\Pr(\omega = L) = \pi \geq \frac{1}{2}$
- Each voter i receives a signal $s_i \in \{l, r\}$,
  private info., conditionally i.i.d. draws with precision $\gamma > \pi$

Model: Preferences

- Each voter i also draws a *preference-type*, $t_i \in \{L_p, N, R_p\}$, private info., i.i.d., prob. $\tau_L \in (0, 0.5)$, $\tau_R \in (0, 0.5)$

- State-independent preferences for *L*-partisans and *R*-partisans

$$
\begin{aligned}
u(L_p, W, \omega) &= \mathbf{1}_{\{W=L\}} \text{ for } \omega \in \{L, R\} \\
u(R_p, W, \omega) &= \mathbf{1}_{\{W=R\}} \text{ for } \omega \in \{L, R\}
\end{aligned}
$$

- Neutrals desire to match winner to state

$$
\begin{aligned}
u(N, L, L) &= u(N, R, R) = 1 \\
u(N, L, R) &= u(N, R, L) = 0
\end{aligned}
$$

Model: Some Definitions

- A sequential voting game is $G\left(\pi, \gamma, \tau_L, \tau_R; n\right)$
- History for i is $h^i \in \{L, R\}^{i-1}$
- Pure strategy for i is
  $v_i : \{L_p, N, R_p\} \times \{L, R\}^{i-1} \times \{l, r\} \to \{L, R\}$
  - *Informative voting* if $l \to L$, $r \to R$
- Beliefs $\mu_i(h^i, s_i)$
- Perfect Bayesian Equilibrium

PBV: Definition

A strategy profile is Posterior-Based Voting if ever voter

- votes “sincerely” for the candidate she believes to be better for her at the time of voting;
- looks only backward to extract information;
- does not condition on being pivotal/influencing future voters;
- if indifferent, votes informatively.

(This should appear myopic and not obviously rational.)

Formal Definition

PBV: Behavior

In PBV,

1. A Partisan votes her bias

2. A Neutral votes

$$v_i(N, h^i, s_i) = \begin{cases} L & \text{if } \mu_i(h^i, s_i) > \frac{1}{2} \\ R & \text{if } \mu_i(h^i, s_i) < \frac{1}{2} \\ \text{informatively} & \text{if } \mu_i(h^i, s_i) = \frac{1}{2} \end{cases}$$

PBV: Example

- Prior $\pi = \frac{1}{2}$, Precision $\gamma = \frac{3}{4}$, and Partisanship $\tau_L = \tau_R = \tau = \frac{1}{4}$
- Voter 1 votes his signal if Neutral since $\gamma > \pi$
- Voter 2:
  - Suppose he has observed L vote
  - Bayes updates on the state: either 1 was L-partisan or Neutral with l signal
  - Votes his signal if Neutral
- Voter 3:
  - Suppose he has observed two L votes
  - Bayes updates on the state
  - Yet, votes her signal if Neutral

PBV: Example

- Voter 4:
  - Suppose he has observed three L votes
  - Bayes updates on the state: $\mu_3(\omega = L | h^4 = LLL, r) > \frac{1}{2}$
  - If Neutral, ignores signal and votes L

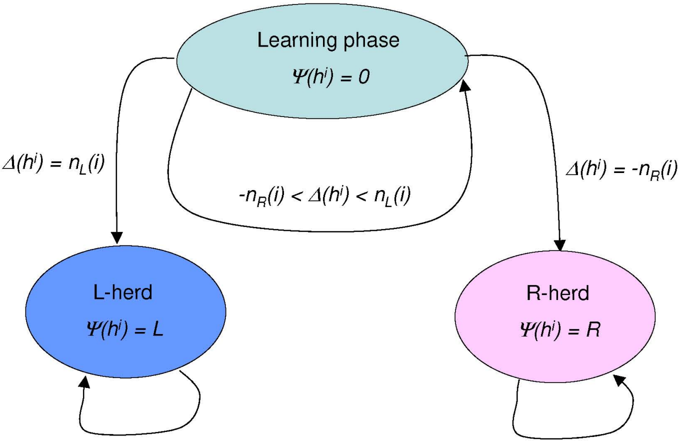
- Voter 5:
  - Realizes that voter 4's vote contains no information
  - So is in the same shoes as voter 4 (as far as PBV goes)
  - Votes L if Neutral

$\Rightarrow$ Voters $4, 5, \ldots$ are in a herd/bandwagon once $h^4 = LLL$

- General points to note:
  1. Bandwagons do not mean that election is decided
  2. Partisans add noise: as $\tau \uparrow$, slower herding
  3. Similar logic for $\tau_L \neq \tau_R$: judging relative to expectations

PBV: General Dynamic System

- Vote lead (for L): $\Delta(h^i)$
- Construct herding thresholds $n_L(i)$ and $-n_R(i)$
- Phase map: $\Psi(h^i) \in \{L, 0, R\}$



PBV: Main Result

PBV entails social learning and can generate momentum, but is this rational for strategic agents?

Theorem

For every game, $G(\pi, \gamma, \tau_L, \tau_R; n)$, the PBV strategy profile is an equilibrium.

PBV: Equilibrium Proof Sketch

Must show: for each i, it is optimal for

1. Partisan i to vote her bias

2. Neutral i to herd in a herding phase

3. Neutral i to vote informatively in the learning phase

PBV: Equilibrium Proof Sketch

Must show: for each i, it is optimal for

1. Partisan i to vote her bias
   - In PBV, future votes are weakly monotonic in history
2. Neutral i to herd in a herding phase
3. Neutral i to vote informatively in the learning phase

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1. Partisan i to vote her bias
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2. Neutral i to herd in a herding phase
   - Once herding starts, no further useful information revealed
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2. Neutral i to herd in a herding phase
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PBV: Equilibrium Proof Sketch

- Is it optimal for Neutral i to vote informatively if in the learning phase?
  - Benefit: voting for who she thinks is currently better, and revealing info
  - Cost: Pushing future voters closer to herding; pushing election closer to ending
- Lots of IC's to check: each voter, each possible vote lead
- Can show that IC is tightest for "penultimate voter"

Skip Example

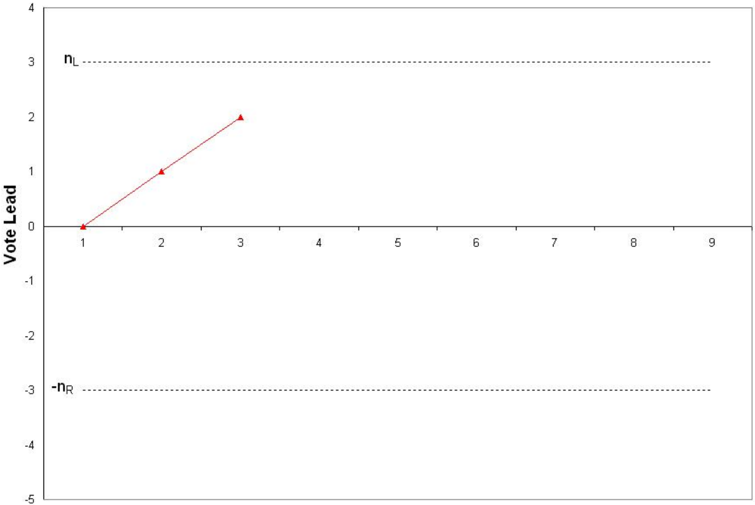
PBV: Equilibrium in the Example

Recall the example earlier:

- Prior $\pi = \frac{1}{2}$, Precision $\gamma = \frac{3}{4}$, and Partisanship $\tau_L = \tau_R = \tau = \frac{1}{4}$
- The thresholds for the herding phase to begin is a vote lead of 3 for either candidate
- Consider voter 3 with $h^3 = LL$ and signal l: he should vote L and trigger the herd, but can he instead profit by deviating?

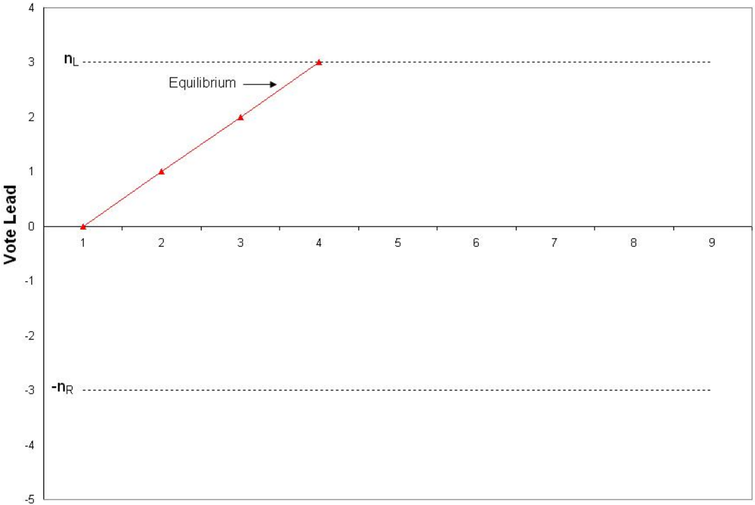
PBV: Equilibrium in the Example

Voter 3 gets an *l* signal, sees vote lead of 2



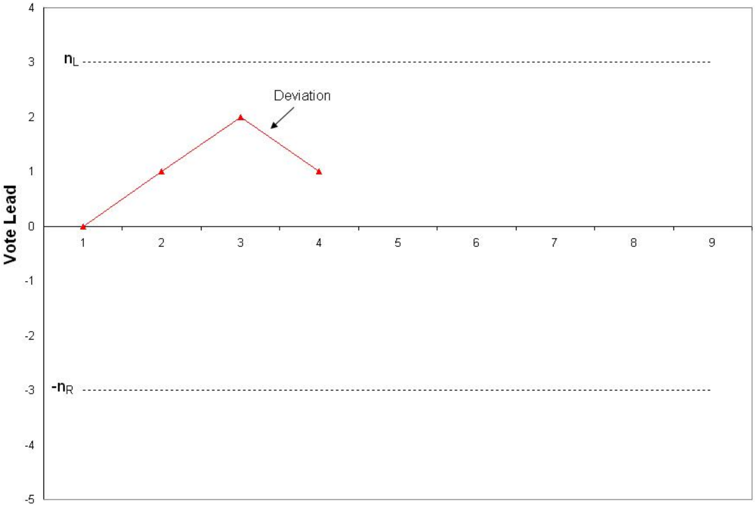
PBV: Equilibrium in the Example

She should vote L and trigger an L herd



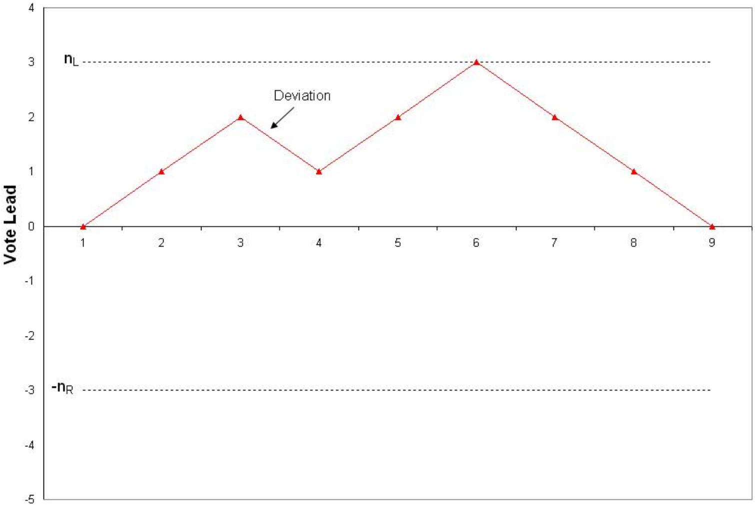
PBV: Equilibrium in the Example

Voter 3 could deviate



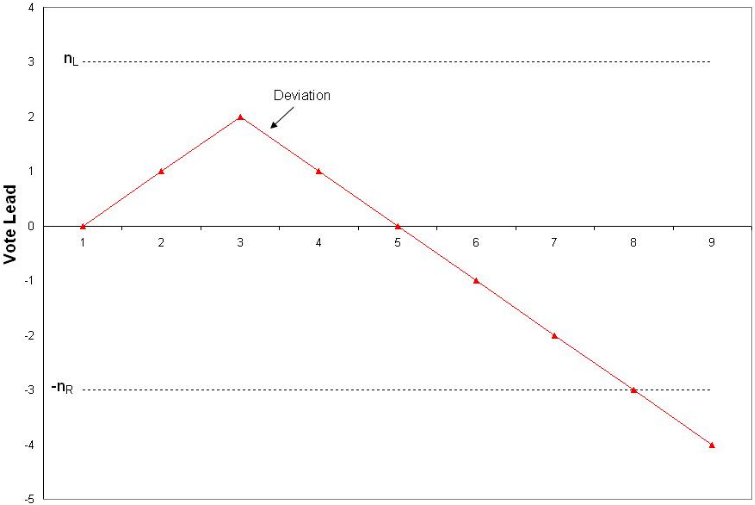
PBV: Equilibrium in the Example

An *L* herd could ensue after deviation



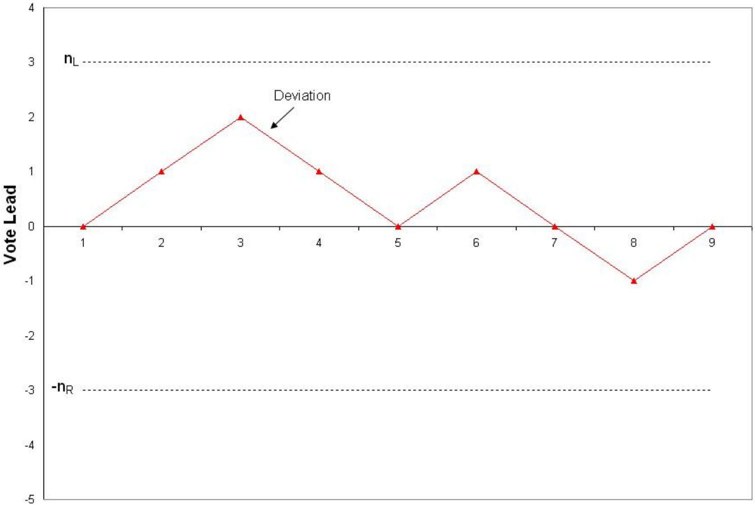
PBV: Equilibrium in the Example

An *R* herd could ensue after deviation



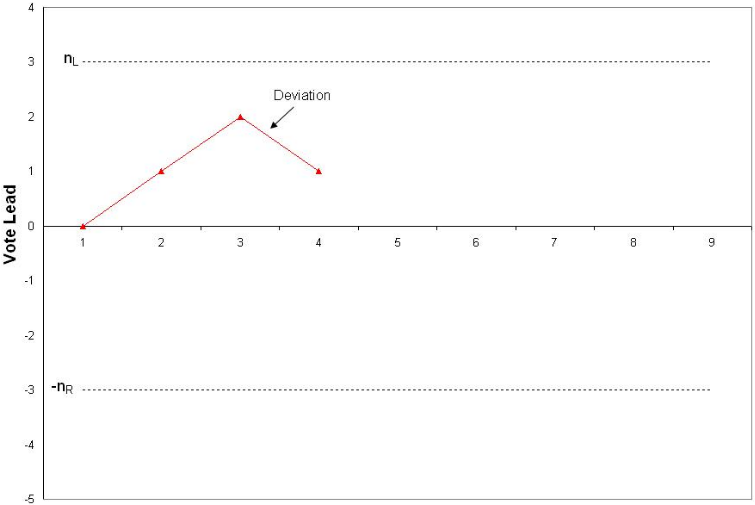
PBV: Equilibrium in the Example

No herd could ensue after deviation



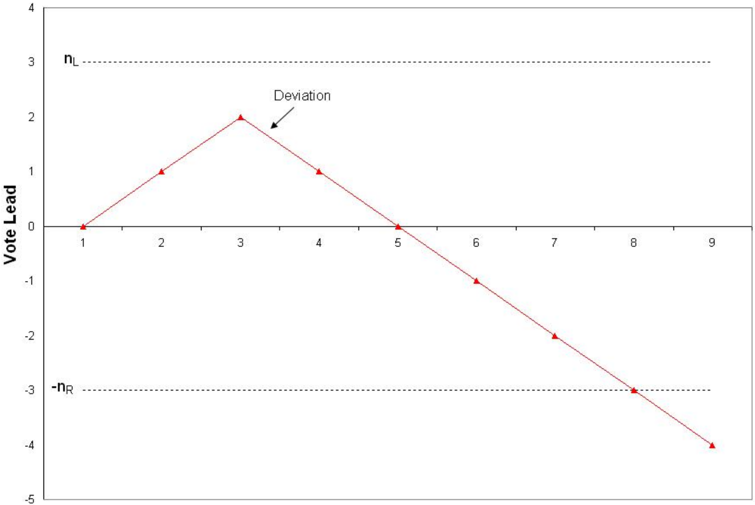
PBV: Equilibrium in the Example

Most attractive scenario for deviation?



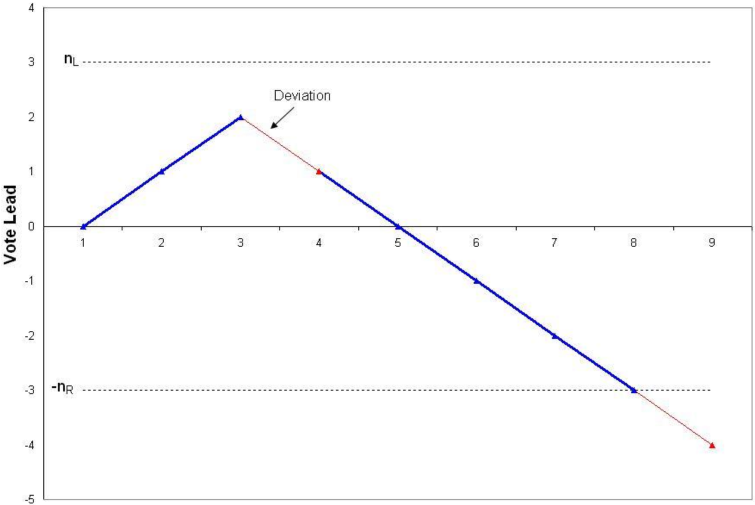
PBV: Equilibrium in the Example

Biggest incentive to deviate if R herd would ensue



PBV: Equilibrium in the Example

But even then, no incentive to deviate



Strict Equilibrium

Definition

An equilibrium is strict if a voter does strictly worse by deviating at any *undecided history*.

- In the spirit of usual strictness
- At decided history, voter is always indifferent

Theorem

PBV is generically a strict equilibrium.

Implications:

1. Not based on convenient resolution of indifference
2. Robust to small perturbations of game

Other Voting Rules

- Can generalize our results to arbitrary monotonic voting rules, e.g. the class of all quota rules with status quo B
- Equivalence of quota rules in large elections

Theorem

1. *PBV is an equilibrium for any monotone voting rule.*
2. *Given two quota rules with $q, q' \in (\tau_L, 1 - \tau_R)$, and any $\varepsilon > 0$, there exists $\bar{n}$ such that for all $n > \bar{n}$, if voters play PBV then*

$$|Pr(L \text{ wins under } q\text{-rule }) - Pr(L \text{ wins under } q'\text{-rule})| < \varepsilon$$

Comparative Statics

Judging relative to expectations $\implies$ bigger partisan base not necessarily better

- direct vs. indirect (social learning) effect

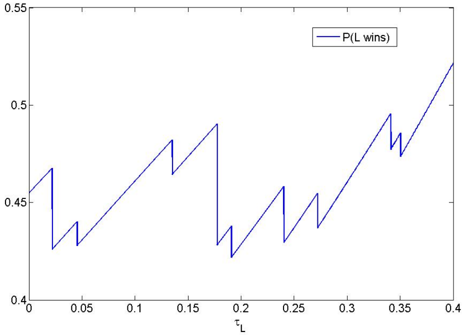


Figure: Probability that L wins as a function of L-partisanship under simple majority rule, $n = 7$, $\gamma = 0.7$, $\pi = 0.6$, and $\tau_R = 0.4$.

Comparative Statics

Judging relative to expectations $\Longrightarrow$ bigger partisan base not necessarily better

- direct vs. indirect (social learning) effect

New York Times, 02/05/08:

> *"But Mr. Obama has suffered one of those external political problems that often madden campaigns: a last-minute California poll that showed him closing in on Mrs. Clinton – in the process, raising expectations that he will win. No wonder Mr. Obama's advisers are suddenly talking about the big surge of early voting in California before Mr. Obama began to break through there."*

History Independent Equilibria

- In simultaneous voting game analog, there is symmetric equilibrium, generically in mixed strategies

- Such history *independent* behavior will remain an equilibrium in our sequential game (DP 2000)

  $\Longrightarrow$ Expectations are key: whether it is optimal to ignore history or not depends on what one expects future voters to do

- This is ultimately an empirical matter

Experiments

Work in progress with Nageeb Ali, Jacob Goeree (Caltech), and Tom Palfrey (Caltech)

Experiments

Work in progress with Nageeb Ali, Jacob Goeree (Caltech), and Tom Palfrey (Caltech)

- Simultaneous and sequential voting treatments in the lab
  - Majority and Unanimity Rules
  - Pure common-values ($\tau_L = \tau_R = 0$)
- Goals:
  1. History dependence vs. independence
  2. Fit of equilibrium theories
- Preview of findings
  - Clear evidence of history dependence
    - Strong momentum under unanimity rule
    - Significant but less sharp momentum under majority rule
  - Need for mixture/noisy equilibrium theory to explain data

Experimental Design

- 2 urns: R and B
  - R contains 2 red balls and 1 blue ball
  - B contains 1 red ball and 2 blue balls

- Urn is selected randomly by computer, uniform prior

Experimental Design

- Subjects are put into groups of $n \in \{3, 6, 12\}$ players
  - Either simultaneous or sequential elections
  - Either unanimity (status quo B) or majority rule (random winner if tied)
- Each group plays 30 rounds, randomized voting order each time if sequential
- In each election, each subject observes 1 ball from urn with replacement, and prior history of votes if any; then votes either R or B
- Payoffs: \$1.00 if group guesses right urn, \$0.10 otherwise

Experimental Parameters: Theory

Focus on $n = 6, 12$

- In the Simultaneous election, unique responsive symmetric BNE (FP, 1998)
  - Majority rule: vote your signal
  - Unanimity rule:
    - if signal r, vote R with probability 1
    - if signal b, vote R with prob. 0.66 if $n = 6$ and prob. 0.83 if $n = 12$
- In the Sequential election
  - the above history-independent behavior is an eqm (DP)
  - but also PBV: for either voting rule and n, in *undecided histories*,
    - vote your signal if vote lead (for R) is $-2 < \Delta < 2$
    - vote for R if $\Delta \geq 2$; vote for B if $\Delta \leq -2$

Data Overview

For the $n = 6$ elections

Timing	Rule	# Groups	# Rounds	All obs.	Und. hist.
Seq	Maj	6	30	1080	916
Sim	Maj	6	30	1080	n/a
Seq	Una	6	30	1080	549
Sim	Una	6	30	1080	n/a

For the $n = 12$ elections

Timing	Rule	# Groups	# Rounds	All obs.	Und. hist.
Seq	Maj	4	30	1440	1111
Sim	Maj	2	30	720	n/a
Seq	Una	6	30	2160	919
Sim	Una	6	30	2160	n/a

Simultaneous Elections: Fraction of votes for R

n=6

signal	Unanimity	Majority
b	.52 (323/616)	.05 (27/597)
r	.94 (437/464)	.96 (463/483)

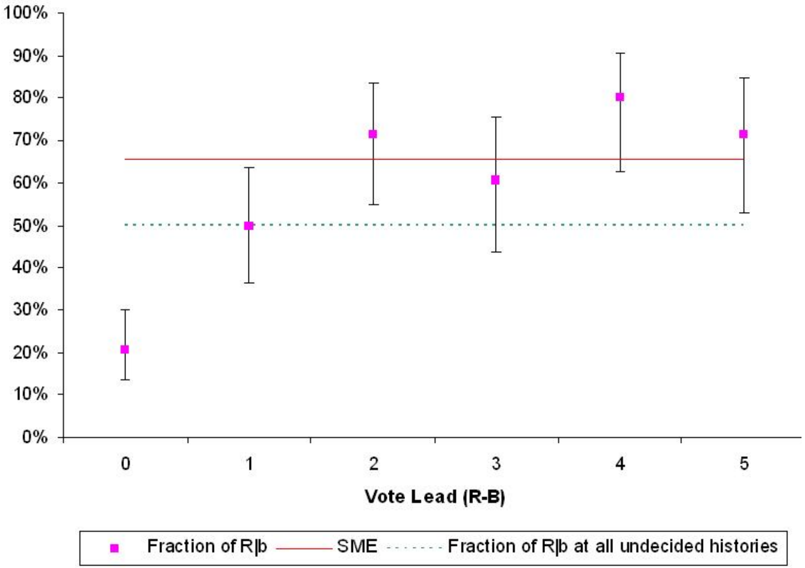
- Recall SME in unanimity $Pr(R|b) = .66$
- Comparable numbers to GMP (APSR, 2000)

n=12

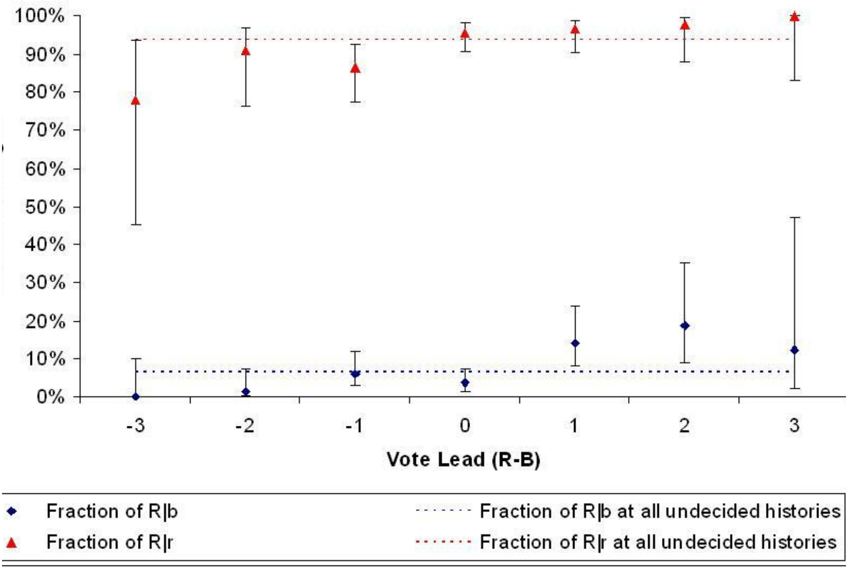
signal	Unanimity	Majority
b	.62 (761/1235)	.05 (20/398)
r	.95 (877/925)	.94 (304/322)

- Recall SME in unanimity $Pr(R|b) = .83$

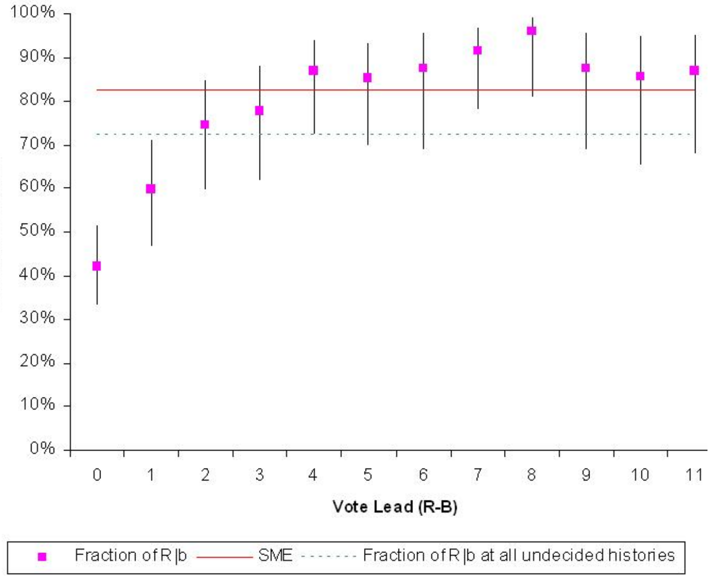
Sequential Elections: $n = 6$, unanimity rule



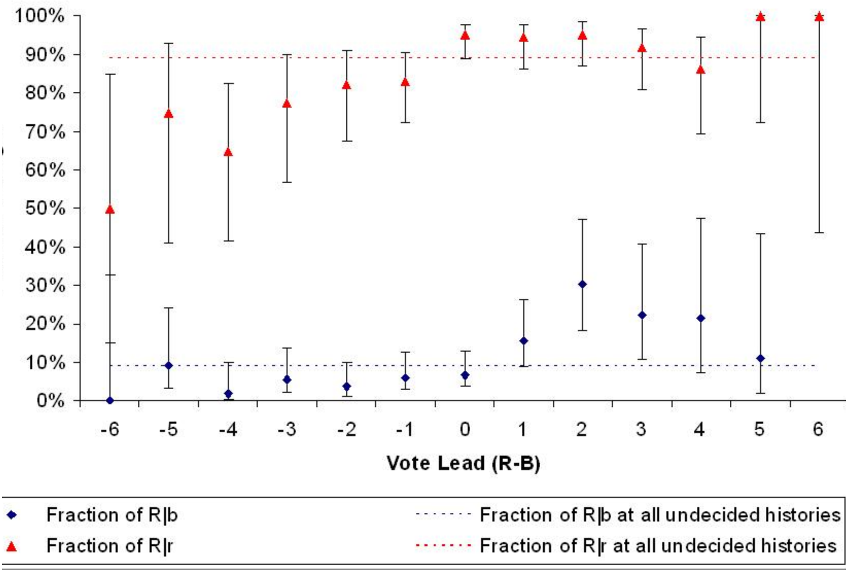
Sequential Elections: $n = 6$, majority rule



Sequential Elections: $n = 12$, unanimity rule



Sequential Elections: $n = 12$, majority rule



History Dependence: A Statistical Test

- p-values of Likelihood-ratio test that fraction of R votes is constant across positions

	n=6	n=12
R|b in Una	0.00	0.00
R|b in Maj	0.00	0.00
R|r in Maj	0.02	0.00

- Above calculation assumes alternate hypothesis, H_a, is non-constancy, but the same point would hold if H_a is (weak) monotonicity

- History dependence under majority rule is especially striking since SME (informative voting) is efficient and simple there

Sequential Elections: Subject-Level Analysis

Preliminary, focus on Unanimity rule

- There is some heterogeneity in behavior across subjects

  Examples...

- In Seq $n = 6$,
  - one subject voted R every time she received a b signal (6)
  - one voted B every time she received a b signal (12)

- Similarly, in Seq $n = 12$,
  - one subject voted R every time she received a b signal (9)
  - one voted B every time she received a b signal (10)

Sequential Elections: Subject-Level Analysis

- But even at individual subject-level, there is clearly some history dependence
- Fraction of subjects who voted B every time they received a b signal

	All positions	Only Pos. 1
n=6	.10 (2/21)	.56 (18/32)
n=12	.01 (1/72)	.49 (29/59)

- Likelihood ratio test that a subject's fraction of $R|b$ is constant across positions is hard to reject b/c of # data points
- Yet, we reject constancy (at 5% level) for
  - 31% (11/36) subjects in $n = 6$
  - 8% (6/72) subjects in $n = 12$

Fitting the Data

- Neither PBV nor SME can fit the data very well
  - e.g., in Seq Una, $\%R|b$ in pos. 1 is too high for PBV, too low for SME
- Note: because non-generic parameters, tie-breaking is relevant in PBV, but none of the variations fit well either
- Seems plausible that different subjects are playing different equilibrium strategies
- We plan to estimate mixture models and QRE

Conclusion: Summary

Theory

- Simple sequential voting model: social learning and payoff interdependencies
- A foundation for rational herds in elections
  - PBV accommodates sincere and sophisticated voting behavior
  - Momentum effects and surprise victories

Experiments

- Confirm history dependence and momentum effects
- But not as strong as theory would predict

Conclusion: Future Research

- Information acquisition
- Richer information/preference structures
- Multiple candidates
- Richer timing structures

Appendix

PBV: Definition

Informal Definition

A strategy profile, $\mathbf{v}$, is Posterior-Based Voting if
for every voter i, pref-type t_i, history h^i, signal s_i,

1. $v_i(t_i, h^i, s_i) = \arg\max_W \mathbb{E}_\omega[u_i(t_i, W, \omega) | h^i, s_i; \mathbf{v_{j \leq i}}]$ (if singleton)

2. $\mathbb{E}_\omega[u_i(t_i, L, \omega) | h^i, s_i; \mathbf{v_{j \leq i}}] = \mathbb{E}_\omega[u_i(t_i, R, \omega) | h^i, s_i; \mathbf{v_{j \leq i}}]$
   $\Rightarrow v_i(t_i, h^i, s_i) = s_i$

Note about (2):

* Matters only non-generically

* Hostile towards cascades

Note about (1):

* Says nothing about PBV being rational

PBV: Definition

◂ Informal Definition

A strategy profile, **v**, is an Equilibrium if
for every voter i, pref-type t_i, history h^i, signal s_i,

$$v_i(t_i, h^i, s_i) \in \arg\max_c \mathbb{E}_{W,\omega}[u_i(t_i, W, \omega)|h^i, s_i; \mathbf{v_{-i}}, Piv_i, Vote_i = c]$$

Note about (2):
* Matters only non-generically

* Hostile towards cascades

Note about (1):
* Says nothing about PBV being rational

PBV: Definition

◂ Informal Definition

A strategy profile, $\mathbf{v}$, is Posterior-Based Voting if
for every voter i, pref-type t_i, history h^i, signal s_i,

1. $v_i(t_i, h^i, s_i) = \arg\max_W \mathbb{E}_\omega[u_i(t_i, W, \omega)|h^i, s_i; \mathbf{v_{j \leq i}}]$ (if singleton)

2. $\mathbb{E}_\omega[u_i(t_i, L, \omega)|h^i, s_i; \mathbf{v_{j \leq i}}] = \mathbb{E}_\omega[u_i(t_i, R, \omega)|h^i, s_i; \mathbf{v_{j \leq i}}]$
   $\Rightarrow v_i(t_i, h^i, s_i) = s_i$

Note about (2):

* Matters only non-generically

* Hostile towards cascades

Note about (1):

* Says nothing about PBV being rational

* PBV = sincere

Discussion: Pure Common Values

- In PBV equilibrium, all histories are on the path
  $\implies$ beliefs are well-defined everywhere
  ($\implies$ Sequential Equilibrium)

- Limit as $\tau_L, \tau_R \to 0$: $n_L(i) = 1$, $n_R(i) = -2$

- This PBV is an equilibrium of limit game with $\tau_L = \tau_R = 0$

- But in limit game, off-path histories exist

- Our limit belief, "ignore a deviation from a herd", survives *D1, Divinity, etc.*

- Fey (2000) impose an ad-hoc belief restriction to conclude that herding is not rational

Discussion: Likelihood of Herds in PBV

- Assume play is PBV and wlog, the true state is R
- The public likelihood ratio,

$$\lambda\left(h^i\right) := \frac{Pr(\omega = L|h^i)}{Pr(\omega = R|h^i)},$$

  is a stochastic process governed by the draws of preference-types and private signals
- $\langle\lambda_i\rangle$, is a (conditional) martingale

Discussion: Likelihood of Herds in PBV

- By the MCT, $\langle \lambda_i \rangle \stackrel{a.s.}{\to} \lambda_\infty$
- Key observation: public LR cannot settle down unless voting is uninformative about signal

Theorem

For every $(\pi, \gamma, \tau_L, \tau_R)$ and for every $\varepsilon > 0$, there exists $\overline{n} < \infty$ such that for all $n > \overline{n}$, if voters play PBV, then

$$\Pr[\textit{a herd develops in } G(\pi, \gamma, \tau_L, \tau_R; n)] > 1 - \varepsilon$$

Discussion: Population Uncertainty

- In large elections, requiring common knowledge of the number of voters may be unrealistic

- PBV remains an equilibrium for a large class of population-uncertainty variations of our model

  Formally,

  - Nature draws the population size N from a probability measure ν on $\mathbb{N}$; N is unobserved by voters
  - Voter i is selected to vote if and only if $i \leq N$
  - Rest of the game is as before

- Subsumes poisson uncertainty (Myerson) and binomial uncertainty (Feddersen-Pesendorfer)

Aggregate Data: Fraction of votes for R

n=6

	Simultaneous		Sequential (und. hist.)	
signal	Unanimity	Majority	Unanimity	Majority
b	.52 (323/616)	.05 (27/597)	.50 (131/262)	.06 (32/507)
r	.94 (437/464)	.96 (463/483)	.98 (281/287)	.94 (383/409)

- Recall SME in unanimity $Pr(R|b) = .66$
- Comparable numbers to GMP (APSR, 2000)

n=12

	Simultaneous		Sequential (und. hist.)	
signal	Unanimity	Majority	Unanimity	Majority
b	.62 (761/1235)	.05 (20/398)	.72 (340/471)	.09 (57/630)
r	.95 (877/925)	.94 (304/322)	.98 (441/448)	.89 (428/481)

- Recall SME in unanimity $Pr(R|b) = .83$

Sequential Elections: Fraction of votes for R

n=6, undecided histories

signal	Unanimity	Majority
b	.50 (131/262)	.06 (32/507)
r	.98 (281/287)	.94 (383/409)

- Recall SME in unanimity $Pr(R|b) = .66$

n=12, undecided histories

signal	Unanimity	Majority
b	.72 (340/471)	.09 (57/630)
r	.98 (441/448)	.89 (428/481)

- Recall SME in unanimity $Pr(R|b) = .83$