Supplementary Appendix for
"Informative Cheap Talk in Elections"

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This Supplementary Appendix generalizes our paper's main results. Section 1 presents a more general model. Section 2 provides results about how sufficiently strong reputation concerns make the voter worse off when she has uncertainty about the policymaker's (PM) type than under any known type of PM. Section 3 discusses informative cheap talk in the campaign stage. Proofs are in Section 4.

1. Model with Arbitrary Number of Types and Actions

The baseline model in the paper assumed binary type and action spaces and made some assumptions (e.g. related to log convexity) on the distribution of states. We relax these assumptions here.

Setting. Assume the set of possible types for the policymaker is $\Theta \equiv \{\theta_1, \theta_2, \ldots, \theta_T\} \subset \mathbb{R}$, where $T \geq 2$ ($T \in \mathbb{N}$), $\theta_i > \theta_{i-1}$ for all $i \in \{2, \ldots, T\}$, and $0 \in \Theta$. Denote the prior on a politician's type by the vector $\boldsymbol{p} \equiv (p_1, \ldots, p_T)$ where p_i is the probability of type θ_i. For convenience, we also write $p[\theta]$ for the probability that $\boldsymbol{p}$ assigns to type θ. For now we consider only the policymaking stage, assuming that the PM is drawn according to $\boldsymbol{p}$; the campaign stage is discussed in Section 3. The PM chooses an action after observing the state of the world. The set of states is $S \equiv \mathbb{R}$ with cumulative distribution function (CDF) $F(\cdot)$. We

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assume that the distribution has support S with a strictly positive density, $f(\cdot)$.[1] The PM's action must be chosen from a finite set $A \equiv \{a_1, \ldots, a_N\} \subset \mathbb{R}$, where $\underline{a} \equiv a_1 < \cdots < a_N \equiv \overline{a}$ and $N \geq 2$ ($N \in \mathbb{N}$). The PM's preferences over actions $a \in A$ for any $\theta \in \Theta$ and $s \in S$ are represented by the utility function $u(a, s, \theta)$. We assume that for all a, s, and θ,

$$u(a, s - \theta, \theta) = u(a, s, 0). \tag{1}$$

We further assume that $u(\cdot)$ is differentiable in s and that, for all a', a, s, and θ,

$$a' > a \implies \frac{\partial}{\partial s}[u(a', s, \theta) - u(a, s, \theta)] > 0, \tag{2}$$

$$a' > a \implies \begin{cases} \lim\limits_{s \to -\infty} [u(a', s, \theta) - u(a, s, \theta)] = -\infty \\ \lim\limits_{s \to \infty} [u(a', s, \theta) - u(a, s, \theta)] = \infty. \end{cases} \tag{3}$$

Requirement (1) says that the policymaker's type acts like a "location shift"; type 0's preferences are sufficient to deduce those of all other types. Requirement (2) says that the PM's utility function is supermodular in a and s; while (3) says that in extreme states the payoff difference between different actions becomes arbitrarily large. An example that satisfies all the conditions is $u(a, s, \theta) = -(a - s - \theta)^2$, which is the utility function used in the baseline model.

We assume that the voter shares the policy preferences of a type-0 PM: the voter's utility function is $u_v(a, s) := u(a, s, 0)$. As in the baseline model the voter updates her belief about the PM after observing his action, but does not observe the state or her own utility.

The PM cares about reputation in addition to policy. A posterior $\hat{\boldsymbol{p}}(a) \equiv (\hat{p}_1(a), \ldots, \hat{p}_T(a))$ provides reputational value $kV(\hat{\boldsymbol{p}}(a))$, where $k > 0$ reflects the strength of reputation concern and $V(\cdot)$ is a continuous function that maps voter posteriors into real numbers. We assume, as a normalization consistent with the baseline model, that $\min\limits_{\boldsymbol{p} \in \Delta\Theta} V(\boldsymbol{p}) = 0$ and $\max\limits_{\boldsymbol{p} \in \Delta\Theta} V(\boldsymbol{p}) = 1$. The PM's net payoff is given by $u(a, s, \theta) + kV(\hat{\boldsymbol{p}}(a))$.

Strategies and Equilibria. We say that a PM's pure strategy is *monotonic* if, for all θ, $s < s'$ implies that the action taken after s is no higher than the action taken after s'. Let $\boldsymbol{\delta_\theta}$ denote the probability distribution that puts probability one on type θ. We say that a distribution

[1] Strictly speaking, this is not a generalization of the baseline model because there we allowed for a finite lower bound on states. As should be clear from the subsequent analysis, we could also allow that here if we restricted attention to $\theta \geq 0$ (as in the baseline model).

over types $\boldsymbol{p}$ is non-degenerate if it has non-singleton support. Without any essential loss of generality (because of the properties of $u(\cdot)$), we focus on equilibria in pure strategies in which, if some type takes some action on path, it takes this action for a positive-probability set of states.

For any given posterior belief function $\hat{\boldsymbol{p}}(\cdot)$ we define

$$\Delta(a', a, s, \theta) := u(a', s, \theta) + kV(\hat{\boldsymbol{p}}(a')) - [u(a, s, \theta) + kV(\hat{\boldsymbol{p}}(a))]$$

as the utility difference for the PM between choosing a' and a when the voter updates using $\hat{\boldsymbol{p}}(\cdot)$. Plainly, (2) and (3) imply that for any a', a, s, θ, and $\hat{\boldsymbol{p}}(\cdot)$,

$$a' > a \implies \frac{\partial}{\partial s}\Delta(a', a, s, \theta) > 0, \tag{4}$$

$$a' > a \implies \begin{cases} \lim_{s \to -\infty} \Delta(a', a, s, \theta) = -\infty \\ \lim_{s \to \infty} \Delta(a', a, s, \theta) = \infty. \end{cases} \tag{5}$$

It follows from (4) and (5) that in any equilibrium the PM uses a monotonic strategy and every types takes actions $\overline{a}$ and $\underline{a}$.

Let

$$\mathcal{S} := \{\boldsymbol{s} : s_1 = -\infty < s_2 \leq \cdots \leq s_N < s_{N+1} = +\infty\}.$$

We can identify the PM's behavior in any equilibrium by a vector for each type, $\boldsymbol{s}(\theta, \boldsymbol{p}, k) \in \mathcal{S}$, with the understanding that type θ plays action a_n if (and only if, modulo behavior at the boundaries) $s \in (s_n(\theta, \boldsymbol{p}, k), s_{n+1}(\theta, \boldsymbol{p}, k))$. If $s_n(\theta, \boldsymbol{p}, k) = s_{n+1}(\theta, \boldsymbol{p}, k)$ then type θ does not play a_n. Since both $\underline{a}$ and $\overline{a}$ are taken with positive probability by all types, it holds in any equilibrium and for any type that $s_1 < s_2 \leq s_N < s_{N+1}$. A type only takes actions $\underline{a}$ and $\overline{a}$ if and only if it uses $\boldsymbol{s}$ with $s_1 = s_N$.

Equation 1 permits a simplification: in any equilibrium, any type θ will use cutoffs that differ from type 0's cutoffs by θ. This property follows from the fact that, for any a', a, s, θ, $\Delta(a', a, s - \theta, \theta) = \Delta(a', a, s, 0)$, and so, given any updating rule for the voter, the preferences of type θ at state $s - \theta$ are identical to those of type 0 at state s. Hence, in any equilibrium $\boldsymbol{s}(\theta, \boldsymbol{p}, k)$, it holds that for all θ and n:

$$s_n(\theta, \boldsymbol{p}, k) = s_n(0, \boldsymbol{p}, k) - \theta. \tag{6}$$

By Equation 6 any equilibrium can be characterized by a single vector $\boldsymbol{s}(\boldsymbol{p},k) := \boldsymbol{s}(0,\boldsymbol{p},k) \in \mathcal{S}$, which we will also view and refer to as a partition of S. For any $\boldsymbol{p}$ and k, an equilibrium exists by Milgrom and Weber (1985, Theorem 1). Moreover, standard arguments (or applying Milgrom and Weber (1985, Theorem 2)) imply that the equilibrium correspondence is upper hemi-continuous in $\boldsymbol{p}$ and k. We define $\mathcal{U}(\boldsymbol{p},k)$ to be the voter's expected utility when the PM is drawn according to the prior $\boldsymbol{p}$ and the reputation concern is k; it is important to note that left implicit is an equilibrium selection $\boldsymbol{s}(\boldsymbol{p},k)$. Taking a selection $\boldsymbol{s}(\boldsymbol{p},k)$ that is continuous in $\boldsymbol{p}$—as is feasible by the aforementioned upper hemi-continuity property—ensures that $\mathcal{U}(\boldsymbol{p},k)$ is continuous in $\boldsymbol{p}$. We will use this continuity property below.

If $k=0$, the PM has no reputational concern and so chooses a to maximize $u(\theta,a,s)$. Define $\boldsymbol{s}_\theta := \boldsymbol{s}(\boldsymbol{p},0)-\theta$ as the partition that would be used by a PM of type θ in this case. If the PM's type is known then there is no updating by the voter, and hence $\boldsymbol{s}(\boldsymbol{\delta_\theta},k) = \boldsymbol{s}_\theta$ for all k and θ. Hence the voter's expected utility from a known candidate of type θ is $\mathcal{U}(\boldsymbol{\delta_\theta},k) = \mathcal{U}(\boldsymbol{\delta_\theta},0)$.

Assumptions for large distortions. Our aim is to establish that large enough reputation concerns can create extreme distortions. We first require that the voter is better off when faced with any known non-congruent type who has all available actions than a known congruent type who is constrained to not choose one of the extreme actions. To state the assumption formally, let $\mathcal{U}_{\tilde{A}}(\boldsymbol{\delta_0},0)$ denote the voter's ex-ante utility when he is faced with known congruent type who is constrained to only choose actions in the set $\tilde{A}$, i.e. choose $a \in \tilde{A} \subseteq A$.

Assumption 1. *For any* $\theta \in \Theta \setminus \{0\}$, $\mathcal{U}(\boldsymbol{\delta_\theta},0) > \max\{\mathcal{U}_{A\setminus\{\overline{a}\}}(\boldsymbol{\delta_0},0), \mathcal{U}_{A\setminus\{\underline{a}\}}(\boldsymbol{\delta_0},0)\}$.

To interpret Assumption 1, observe that the gain from choosing $\overline{a}$ (resp. $\underline{a}$) over any other action in state s becomes become arbitrarily large as $s \to +\infty$ (resp. $s \to -\infty$) by (4). Thus, Assumption 1 holds when there is enough weight on tail states relative to the other parameters. In particular, for any given $F(\cdot)$ and A, it always holds if $|\theta|$ is small enough for all $\theta \in \Theta$. Moreover, when the type and action space are binary and the utility functions are negative quadratic, Assumption 1 is equivalent to the assumption that $\mathbb{E}\left[s|s > \frac{\underline{a}+\overline{a}}{2} - b\right] > \frac{\underline{a}+\overline{a}}{2}$, which was a maintained assumption in the baseline model.

We will also assume that the policymaker receives a higher reputational payoff from being thought more moderate:

Assumption 2. $V(\hat{\boldsymbol{p}}) > V(\hat{\boldsymbol{p}}')$ *if*

(i) $\sum_{i=j}^{m} \hat{p}_i \geq \sum_{i=j}^{m} \hat{p}'_i$ *for all* $j,m \in \{1,\ldots,T\}$ *with* $\theta_j \leq 0 \leq \theta_m$, *and*

(ii) $\sum_{i=j}^{m} \hat{p}_i > \sum_{i=j}^{m} \hat{p}'_i$ *for some* $j, m \in \{1, \ldots, T\}$ *with* $\theta_j \leq 0 \leq \theta_m$.

In general, Assumption 2 does not require that $V(\hat{\boldsymbol{p}}) > V(\hat{\boldsymbol{p}}')$ when $\hat{p}[0] > \hat{p}'[0]$. But when $T = 2$, Assumption 2 simplifies to $\hat{p}[0] > \hat{p}'[0] \iff V(\hat{\boldsymbol{p}}) > V(\hat{\boldsymbol{p}}')$: the PM's reputational payoff is increasing in the perceived probability of being congruent, as in the baseline model.

2. Any Known Devil is Better than an Unknown Angel

This section addresses when any known, biased PM is preferred to a PM with uncertain type in any equilibrium, which we will refer to as "any known devil is better than an unknown angel".

2.1. Asymmetric Distribution of Types

For any probability distribution over types $\boldsymbol{p}$ define

$$p^+(\boldsymbol{p}) := \sum_{\{i:\theta_i>0\}} p_i \text{ and } p^-(\boldsymbol{p}) := \sum_{\{i:\theta_i<0\}} p_i$$

as the probability assigned by $\boldsymbol{p}$ to (strictly) upward and downward biased types respectively.

The following result says that for any sufficiently asymmetric $\boldsymbol{p}$, any known devil is preferred to an unknown angel in every equilibrium.

Proposition 1. *Let Assumption 1 and Assumption 2 be satisfied. For any* $\alpha, \beta > 0$*, there exists an* $\varepsilon > 0$ *such that, for all* $\boldsymbol{p}$ *with* $p[0] \geq \beta$ *and either*

(i) $p^+(\boldsymbol{p}) \geq \alpha$ *and* $p^-(\boldsymbol{p}) < \varepsilon$*, or*

(ii) $p^-(\boldsymbol{p}) \geq \alpha$ *and* $p^+(\boldsymbol{p}) < \varepsilon$*,*

there exists a $\hat{k}(\boldsymbol{p})$ *such that, for all* $k > \hat{k}(\boldsymbol{p})$ *and all* $\theta \in \Theta$*,* $\mathcal{U}(\boldsymbol{p}, k) < \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ *in every equilibrium.*

Here is the intuition behind Proposition 1. If the PM could only be biased in favor of high (resp., low) actions, then owing to Assumption 2, the PM's reputation would always be higher (resp., lower) after choosing $\underline{a}$ as opposed to $\overline{a}$. Consequently, when the type distribution is sufficiently skewed toward the PM having a bias in only direction, one of the extreme actions

$a \in \{\underline{a}, \overline{a}\}$ will only be taken in arbitrarily extreme states as reputational concerns become paramount. By Assumption 1, if the concern for reputation is sufficiently strong, the voter will prefer a PM of known type to an uncertain PM.

Proposition 1 generalizes the message from our baseline model because, when $T = 2$, either $p^-(\cdot) = 0$ or $p^+(\cdot) = 0$, and hence Proposition 1 simply concludes that for any non-degenerate prior, a sufficiently strong reputation concern causes the voter to prefer a known-type PM to the PM of unknown type. Note that even within the scope of $T = 2$, the conclusion here is more general than in the baseline model in multiple ways: the utility functions need not be quadratic; there can be more than two actions; and we allow for any full-support distribution of states. In particular, while the baseline model's assumption about $f(\cdot)$ being log-convex on a suitable domain is needed to ensure a unique equilibrium in the policymaking stage and to obtain clean comparative statics there, it is not necessary for the voter's welfare to be non-monotonic.

2.2. Symmetric Environments

We now provide a result for symmetric environments (in a sense made precise below). Intuitively, when Assumption 2 holds and there are more than two actions ($N > 2$), the PM has an incentive to choose moderate actions in order to signal that he is a moderate type. However, with appropriate off-path beliefs, it is possible to support an equilibrium in which only the extreme actions $\underline{a}$ and $\overline{a}$ are taken. Our next assumption says that it is not in the voter's interest to have the PM only take actions $\underline{a}$ and $\overline{a}$. Specifically, the voter is better off under any known non-congruent PM who can choose any action than under a known congruent PM who is constrained to only choose an extreme action.

Assumption 3. *For all $\theta \in \Theta \setminus \{0\}$, $\mathcal{U}(\boldsymbol{\delta_\theta}, 0) > \mathcal{U}_{\{\underline{a}, -\overline{a}\}}(\boldsymbol{\delta_0}, 0)$.*

Plainly, Assumption 3 can only be satisfied if $N > 2$. If $N > 2$, Assumption 3 holds when the highest and lowest actions are sufficiently far apart (so that intermediate actions are optimal for a broad range of states), or when $|\theta|$ is sufficiently small for all $\theta \in \Theta$. Recall that the latter is also sufficient to guarantee Assumption 1.

We say that the *type space is symmetric* if, for all $i = 1, \ldots, \frac{T-1}{2}$, $\theta_i = -\theta_{T+1-i}$. Further, we say that a *type distribution $\boldsymbol{p}$ is symmetric* if, for all $i = 1, \ldots, \frac{T-1}{2}$, $p_i = p_{T+1-i}$. Finally, the reputation function is symmetric if it only depends on the magnitude, not the direction, the policymaker is thought to be biased. More precisely, given a symmetric type space, *the reputation function is symmetric* if $V(\hat{\boldsymbol{p}}) = V(\hat{\boldsymbol{p}}')$ for all $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{p}}'$ such that $\hat{p}_i + \hat{p}_{T+1-i} = \hat{p}'_i + \hat{p}'_{T+1-i}$ for all $i = 1, \ldots, \frac{T-1}{2}$.

Assumption 4. *The type space, Θ, and reputation function, $V(\cdot)$, are symmetric. The state density, $f(\cdot)$, is symmetric around 0 and single-peaked.*

The following result says that under our assumptions, any known devil is preferred to an unknown angel in any equilibrium of a sufficiently symmetric environment with large enough reputation concerns. The intuition is that, in any equilibrium with large k, either only the extreme actions are ever taken (and the result follows from Assumption 3) or moderate actions are taken for all but the most extreme states (and the result follows from Assumption 1).

Proposition 2. *Let Assumptions 1–4 be satisfied. For any symmetric prior $\boldsymbol{p}$ that has full support, there exists $\varepsilon > 0$ such that for any prior $\boldsymbol{q}$ with $\max_{j\in\{1,\ldots,T\}} |p_j - q_j| < \varepsilon$, there exists $\hat{k}(\boldsymbol{q})$ such that, for all $k > \hat{k}(\boldsymbol{q})$ and all $\theta \in \Theta$, in every equilibrium $\mathcal{U}(\boldsymbol{q}, k) < \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$.*

Combining Proposition 1 and Proposition 2 we see that the generalized distortion result holds, for any number of types, when either: (1) the distribution of types is sufficiently asymmetric (2) the environment is close enough to symmetric. These conditions are sufficient, but not necessary, to generate any known devil being preferred to a unknown angel.

3. Informative Cheap Talk

In this section we study cheap-talk communication during an election between two candidates that determines which of them becomes the PM. Each candidate's type is drawn independently from the prior distribution $\boldsymbol{p}$; they make simultaneous costless and non-binding announcements; and the voter elects one of them.

3.1. A Limiting Case

Proposition 1 and Proposition 2 provide conditions under which the voter prefers a known, biased type to a PM whose type is uncertain. As in the paper's baseline model this creates an avenue for informative cheap-talk communication in the campaign stage. However, a general analysis of candidates' incentives to reveal information in the campaign stage is rather complicated when we have more than two types and actions. For this reason we focus on a limiting case, based on Section 4.2 of the main text, in which candidates care only about winning office in the campaign stage. Once in office, the benefits from being elected the first time is no longer relevant, and the PM's incentives are as in Section 1. While there may not be a unique equilibrium in the policymaking stage we fix a continuous selection of equilibria $\boldsymbol{s}(\boldsymbol{p}, k)$, and consider candidate statements in the campaign stage.

As in the main text's two-type model, we will focus on symmetric equilibria in which both candidates (conditional on type) randomize with the same probability, and the voter's election probability after each pair of messages doesn't depend on which candidate made the announcement. An equilibrium is *informative* if there are distinct messages, each sent with positive probability on the equilibrium path, that induce different beliefs for the voter.

Proposition 3. *Suppose that candidates seek to maximize their probability of election during the campaign stage, and fix a continuous selection of equilibrium, $\boldsymbol{s}(\boldsymbol{p}, k)$, in the policymaking stage. If $\mathcal{U}(\boldsymbol{p}, k) < \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ for some $\theta \neq 0$ then there exists an equilibrium with informative cheap-talk campaigns.*

The requirement that $\mathcal{U}(\boldsymbol{p}, k) < \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ for some $\theta \neq 0$ amounts to saying that *some* known devil is preferred to an unknown angel; it is weaker than saying that *any* known devil is. Plainly, it is sufficient that the hypotheses of either Proposition 1 or Proposition 2 are satisfied.

Proposition 3 derives from an application of the intermediate value theorem. Suppose there are two messages, 0 and θ. If only the type θ candidate ever announced θ (and did so with small enough probability) the voter's utility would be higher from the candidate who announced θ than one who announced 0. Conversely, if all types other than 0 announced θ then the voter would get a higher payoff from the candidate who announced 0. So there exists some intermediate profile of announcements for which the voter is indifferent. When the voter is indifferent and randomizes with equal probability after observing different messages we have an equilibrium. This equilibrium is informative as the posterior is different across different messages: in particular, if the voter observes message θ she knows with certainty the candidate is not type 0.

3.2. Additional Examples

When candidates are forward looking in the campaign stage, they are also concerned with the reputation with which they are elected. As in the baseline model the candidate's utility is

$$c + v_\theta + u(a, s, \theta) + kV(\hat{p}),$$

if elected, with $v_\theta := -\mathbb{E}[u(a, s, \theta)|\boldsymbol{s}_\theta]$, and 0 if not elected. While a general analysis appears intractable, there are salient cases with more than two types/actions for which informative communication exists in the campaign stage when c and k are sufficiently large. Below, we describe the intuitions for two of them; a formal analysis is available on request.

Asymmetric environment. Informative communication is possible when there are two actions and three candidate types with biases in opposite directions (recall that one type is congruent) and a sufficiently asymmetric type distribution. If the probability that candidates are biased towards low (high) actions is sufficiently small, an equilibrium exists in which type-0 and low (high) candidates announce the they are centrist, and high (low) types randomize between revealing their type and claiming to be centrist. The randomization that makes the voter indifferent will reduce, but not eliminate, pandering in office after a claim of congruence, and so the voter will update negatively after observing the high (low) action. Consequently, candidates who are biased towards the high (low) action have less incentive to claim to be centrist, and the incentives are created for informative communication in the campaign. In this sense, the baseline model's results on informative communication are robust to introducing a small probability the PM could be biased in the opposite direction.

Symmetric environment. Another interesting case is a symmetric environment with three types and three actions, $A = \{-1, 0, 1\}$. Here, an equilibrium exists in which type 0 announces that it is type 0 while biased types randomize between announcing their true type and claiming to be type 0. The biased types randomize so that the voter is indifferent between candidates who claim to be centrist and candidates who reveal themselves to be biased. In the policymaking stage, since the voter's belief is symmetric after a claim to be centrist, she will update positively after seeing centrist actions. Biased types then have less incentive to claim to be centrist. So, rather than simply announcing whether they are biased or not, it is possible that candidates sometimes reveal the direction of their bias in their campaigns.

4. Proofs

4.1. Proofs for Section 2

To prove Proposition 1 and Proposition 2, we will first establish a result (Lemma 1 below) that assures that if certain conditions are satisfied and reputational concerns are sufficiently strong, any known devil is preferred to an unknown angel. The conditions (Condition 1 and Condition 2 below) require that for a class of PM's strategies, the voter's posterior should not be constant over on-path actions. We then prove the propositions by verifying that the proposition's hypotheses imply the conditions.

For any $L \geq 0$, let $\mathcal{U}^L$ be the maximum possible expected utility for the voter when she can choose any strategy for the PM subject to a constraint that for all $s \in [-L, L]$, either (i) action $\underline{a}$ must not be taken, or (ii) action $\overline{a}$ must not be taken. Under Assumption 1, continuity implies that there exists $L > 0$ such that the voter's payoff is higher with any known biased type than a known congruent type who is subject to the above constraint, i.e.

$$\forall \theta \in \Theta : \mathcal{U}(\boldsymbol{\delta_\theta}, 0) > \mathcal{U}^L. \tag{7}$$

We let $L > 0$ be some value that satisfies (7) and define

$$\mathcal{S}^L := \{\boldsymbol{s} \in \mathcal{S} : s_2, s_N \in [-L, L]\}.$$

Any strategy in $\mathcal{S}^L$ entails type 0 taking action $\underline{a}$ for all $s < -L$ and action $\overline{a}$ for all $s > L$.

Condition 1. *Suppose Assumption 1 is satisfied and fix a prior $\boldsymbol{p}$. Given any vector $\boldsymbol{s} \in \mathcal{S}^L$, were each type θ to take action a_n if and only if $s \in (s_n - \theta, s_{n+1} - \theta)$, the posterior induced by the on-path actions, $\hat{\boldsymbol{p}}(a)$, would not lead to a constant $V(\hat{\boldsymbol{p}}(a))$ across the on-path a's.*

Condition 1 requires that $V(\cdot)$ be non-constant over on-path actions given any $\boldsymbol{s} \in \mathcal{S}^L$, including those with only two on-path actions (i.e. with $s_2 = s_N$). The next condition weakens the requirement to only those $\boldsymbol{s} \in \mathcal{S}^L$ that have at least three actions taken (i.e. with $s_2 < s_N$).

Condition 2. *Suppose Assumption 1 is satisfied and fix prior $\boldsymbol{p}$. Given any vector $\boldsymbol{s} \in \mathcal{S}^L$ with $s_2 < s_N$, were each type θ to take action a_n if and only if $s \in (s_n - \theta, s_{n+1} - \theta)$, the posterior induced by the on-path actions, $\hat{\boldsymbol{p}}(a)$, would not lead to a constant $V(\hat{\boldsymbol{p}}(a))$ across the on-path a's.*

The key result is:

Lemma 1. *Fix any prior $\boldsymbol{p}$. Suppose either*

1. *Assumption 1 and Condition 1 are satisfied; or*
2. *Assumption 1, Assumption 3, and Condition 2 are satisfied.*

Then there exists a $\hat{k}(\boldsymbol{p})$ such that $k > \hat{k}(\boldsymbol{p}) \implies \mathcal{U}(\boldsymbol{p}, k) < \min_\theta \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ in any equilibrium.

Proof. Part 1: First consider the case in which Assumption 1 and Condition 1 are satisfied. Fix any sequence of equilibria of the corresponding games as $k \to \infty$, $\boldsymbol{s}(\boldsymbol{p}, k) \in \mathcal{S}$. We claim that there exists a $\hat{k}$ such that if $k > \hat{k}$ then either $s_2(\boldsymbol{p}, k) < -L + \theta_1$ or $s_N(\boldsymbol{p}, k) > L + \theta_T$. Suppose not, so there exists a sequence $k_m \to \infty$,

$$\forall k_m : -L + \theta_1 \leq s_2(\boldsymbol{p}, k_m) \leq s_N(\boldsymbol{p}, k_m) \leq L + \theta_T. \tag{8}$$

Then, passing to a subsequence of (k_m) if needed and denoting $s_n^\infty := \lim_{k_m \to \infty} s_n(\boldsymbol{p}, k_m)$ for $n \in \{1, \ldots, N+1\}$, it follows that $-L + \theta_1 \leq s_2^\infty \leq \cdots \leq s_N^\infty \leq L + \theta_T$. By Condition 1, under $(s_n^\infty)_{n=1}^{N+1}$, there exist on path actions a and a' such that $V(\hat{\boldsymbol{p}}(a)) > V(\hat{\boldsymbol{p}}(a'))$. Thus, by continuity of all the relevant objects, once k_m is large enough, given that each type θ uses cutoff $\boldsymbol{s} - \theta$, no type of the policymaker will play a' when $s \in (-L, L)$. But this contradicts (8). Hence we cannot have an equilibrium with $\boldsymbol{s} \in \mathcal{S}^L$.

Finally, since $\mathcal{U}^L$ provides an upper bound on the payoff the voter can receive if there is some action $a \in \{\underline{a}, \overline{a}\}$ that is never taken when $s \in [-L, L]$, by (7) we conclude that when $k > \hat{k}, \mathcal{U}(\boldsymbol{p}, k) < \min_\theta \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$.

Part 2: Now consider the case in which Assumption 1, Assumption 3, and Condition 2 are satisfied. First note that, by Assumption 1, Assumption 3 and continuity, there exists a $B > 0$ such that $\min_\theta \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ is higher than the payoff from any $\boldsymbol{s} \notin \mathcal{S}_B^L := \{\boldsymbol{s} \in \mathcal{S}^L : s_N - s_2 \geq B\}$. Fix any sequence of equilibria of the corresponding games as $k \to \infty$, $\boldsymbol{s}(\boldsymbol{p}, k) \in \mathcal{S}$. We claim that there exists a $\hat{k}$ such that if $k > \hat{k}$ then either $\boldsymbol{s}_2(\boldsymbol{p}, k) < -L + \theta_1$, $\boldsymbol{s}_N(\boldsymbol{p}, k) > L + \theta_T$ or $\boldsymbol{s}_N(\boldsymbol{p}, k) - \boldsymbol{s}_2(\boldsymbol{p}, k) < B$. Suppose not, so there exists a sequence $k_m \to \infty$,

$$\forall k_m : -L + \theta_1 \leq s_2(\boldsymbol{p}, k_m) \leq s_N(\boldsymbol{p}, k_m) - B \leq L + \theta_T - B. \tag{9}$$

Then, passing to a subsequence of (k_m) if needed and denoting $s_n^\infty := \lim_{k_m \to \infty} s_n(\boldsymbol{p}, k_m)$ for $n \in \{1, \ldots, N+1\}$, it follows that $-L + \theta_1 \leq s_2^\infty \leq \cdots \leq s_N^\infty - B \leq L + \theta_T - B$. By Condition 2, under $(s_n^\infty)_{n=1}^{N+1}$, there exist on path actions a and a' such that $V(\hat{\boldsymbol{p}}(a)) > V(\hat{\boldsymbol{p}}(a'))$. Thus, by continuity of all the relevant objects, once k_m is large enough, given that each type θ uses cutoff $\boldsymbol{s} - \theta$, no type of the policymaker will play $\boldsymbol{s} \in \mathcal{S}_B^L$. But this contradicts (9). By the definition of $\mathcal{S}_B^L$ we conclude that when $k > \hat{k}, \mathcal{U}(\boldsymbol{p}, k) < \min_\theta \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$. □

To prove Proposition 1, we will also need the following lemma which demonstrates that if all biases are to the right (left), then the reputation from choosing $\underline{a}$ is higher (lower) than from taking action $\overline{a}$. Intuitively, if it is only possible for the PM to be biased in one direction, and if the prior on the PM is non-degenerate, the posterior on the policymaker's type after observing one extreme action must first order stochastically dominates the other.

Lemma 2. *Let $\boldsymbol{p}$ be any non-degenerate prior on Θ and let $V(\cdot)$ satisfy Assumption 2. For any $\boldsymbol{s} \in \mathcal{S}$, $V(\hat{\boldsymbol{p}}(\underline{a})) > V(\hat{\boldsymbol{p}}(\overline{a}))$ if $p[\theta] = 0$ for all $\theta < 0$, and $V(\hat{\boldsymbol{p}}(\underline{a})) < V(\hat{\boldsymbol{p}}(\overline{a}))$ if $p[\theta] = 0$ for all $\theta > 0$.*

Proof. We provide the argument for the case where $p[\theta] = 0$ for all $\theta < 0$; the other case is analogous. Let j be such that $\theta_j = 0$. When all types with positive prob. are non-negative, Assumption 2 implies that $V(\hat{\boldsymbol{p}}) > V(\hat{\boldsymbol{p}}')$ if, for all $m \in \{j, \ldots, T\}$, $\sum_{i=j}^{m} \hat{p}_i \geq \sum_{i=j}^{m} \hat{p}'_i$, with at least one inequality strict. Now note that for any $\boldsymbol{s} \in \mathcal{S}$ and any $m \in \{j, \ldots, T\}$,

$$\hat{p}_m(\underline{a}) = \frac{p_m F(s_1 - \theta_m)}{\sum_{i=j}^{T} p_i F(s_1 - \theta_i)} \text{ and } \hat{p}_m(\overline{a}) = \frac{p_m(1 - F(s_N - \theta_m))}{\sum_{i=j}^{T} p_i(1 - F(s_N - \theta_i))}.$$

Since $F(s - \theta_i)$ is decreasing in i for any s, it holds that for all $\boldsymbol{s} \in \mathcal{S}$ and $m \in \{j, \ldots, T\}$,

$$\sum_{i=j}^{m} \hat{p}_i(\underline{a}) = \frac{\sum_{i=j}^{m} p_i F(s_1 - \theta_i)}{\sum_{i=j}^{T} p_i F(s_1 - \theta_i)} \geq \frac{\sum_{i=j}^{m} p_i(1 - F(s_N - \theta_i))}{\sum_{i=j}^{T} p_i(1 - F(s_N - \theta_i))} = \sum_{i=j}^{m} \hat{p}'_i(\overline{a}),$$

and, if $m < T$, we have equality only if $p_i \in \{0, 1\}$ for all $i \in \{j, \ldots, m\}$. As $\boldsymbol{p}$ is non-degenerate, this inequality is strict for some $m \in \{j, \ldots, T-1\}$. And so, since $V(\cdot)$ satisfies Assumption 2, this implies that $V(\hat{\boldsymbol{p}}(\underline{a})) > V(\hat{\boldsymbol{p}}'(\overline{a}))$. □

Proof of Proposition 1. We prove the result for the case where $p[0] \geq \beta$, $p^+(\boldsymbol{p}) \geq \alpha$, and $p^-(\boldsymbol{p}) < \varepsilon$; the other case is analogous. By Lemma 1, it is sufficient to show that Condition 1 is satisfied, which follows if we can show that $V(\hat{\boldsymbol{p}}(\overline{a})) \neq V(\hat{\boldsymbol{p}}(\underline{a}))$ for all $\boldsymbol{s} \in \mathcal{S}^L$.

Let $\alpha, \beta > 0$ satisfy $\alpha + \beta < 1$. Define P_α^β to be the set of all distributions such that $p[\theta] = 0$ for all $\theta < 0$, $p[0] \geq \beta$, and $p^+ \geq \alpha$. Note that, by Lemma 2, for all $\boldsymbol{p} \in P_\alpha^\beta$ and $\boldsymbol{s} \in \mathcal{S}^L$, $V(\hat{\boldsymbol{p}}(\underline{a})) > V(\hat{\boldsymbol{p}}(\overline{a}))$. Moreover, since P_α^β and $\mathcal{S}^L$ are compact we can define

$$d_\alpha^\beta := \min_{\boldsymbol{p} \in P_\alpha^\beta, \boldsymbol{s} \in \mathcal{S}^L} V(\hat{\boldsymbol{p}}(\underline{a})) - V(\hat{\boldsymbol{p}}(\overline{a})) > 0.$$

Since $V(\cdot)$ is a continuous function on $\Delta\Theta$, and $\Delta\Theta$ is compact, it follows that $V(\cdot)$ is uniformly continuous. Moreover, since the set of priors is compact, $\hat{\boldsymbol{p}}(a)$ is uniformly continuous in the prior $\boldsymbol{p}$ for any a played on the equilibrium path. Let $A(\boldsymbol{s})$ denote the set of actions taken on the equilibrium path given $\boldsymbol{s}$. Then, for any $\boldsymbol{s} \in \mathcal{S}^L$ and any $a \in A(\boldsymbol{s})$, there exists an

$\varepsilon_1(a, \boldsymbol{s}) > 0$ such that

$$|V(\hat{\boldsymbol{p}}) - V(\hat{\boldsymbol{p}}')| < \frac{d_\alpha^\beta}{2}$$

for any priors $\boldsymbol{p}$ and $\boldsymbol{p}'$ for which

$$\max_{i \in \{1,\ldots,T\}} |p_i - p_i'| < \varepsilon_1(a, \boldsymbol{s}).$$

Furthermore, since $A(\boldsymbol{s}) \subseteq A$ is finite and $\mathcal{S}^L$ is compact,

$$\varepsilon := \min_{\boldsymbol{s} \in \mathcal{S}^L} \min_{a \in A(\boldsymbol{s})} \varepsilon_1(a, \boldsymbol{s}) > 0.$$

Consequently, for any $\boldsymbol{p}$ and $\boldsymbol{p}'$ such that $\max\limits_{i \in \{1,\ldots,T\}} |p_i - p_i'| < \varepsilon$, for all $\boldsymbol{s} \in \mathcal{S}^L$,

$$|V(\hat{\boldsymbol{p}}(\overline{a})) - V(\hat{\boldsymbol{p}}'(\overline{a}))| < \frac{d_\alpha^\beta}{2}.$$

Now let $\boldsymbol{s} \in \mathcal{S}^L$ and $\boldsymbol{p}$ be any prior such that $p[0] \geq \beta$, $p^+(\boldsymbol{p}) \geq \alpha$ and $p^-(\boldsymbol{p}) < \varepsilon$. We define $\boldsymbol{p}'$ such that $p'[\theta] = 0$ if $\theta < 0$, $p'[\theta] = p[\theta]$ if $\theta > 0$, and $p'[\theta] = \sum\limits_{\theta \leq 0} p[\theta]$. Then since $\max\limits_{i \in \{1,\ldots,T\}} |p_i - p_i'| < \varepsilon$, $\boldsymbol{p}' \in P_\alpha^\beta$, and $\underline{a}, \overline{a} \in A(\boldsymbol{s})$, it holds that

$$V(\hat{\boldsymbol{p}}(\overline{a})) < V(\hat{\boldsymbol{p}}'(\overline{a})) + \frac{d_\alpha^\beta}{2} \leq V(\hat{\boldsymbol{p}}'(\underline{a})) - \frac{d_\alpha^\beta}{2} < V(\hat{\boldsymbol{p}}(\underline{a})).$$

Hence Condition 1 is satisfied, and so the result follows from Lemma 1. $\square$

The following Lemma will facilitate the proof of Proposition 2.

Lemma 3. *Under Assumptions 1-4, Condition 2 is satisfied when the prior* $\boldsymbol{p}$ *is symmetric and has full support.*

Proof. For any s_2 and s_N, with $s_2 < s_N$, there are at least three actions $\underline{a}$, a_n, and $\overline{a}$ that are taken on the equilibrium path. Upon observing action $a \in \{\underline{a}, a_n, \overline{a}\}$ the updated beliefs are

$$\hat{p}_j(\underline{a}) = \frac{p_j F(s_2 - \theta_j)}{\sum_{i=1}^{T} p_i F(s_2 - \theta_i)},$$

$$\hat{p}_j(a_n) = \frac{p_j(F(s_{n+1} - \theta_j) - F(s_n - \theta_j))}{\sum_{i=1}^{T} p_i(F(s_{n+1} - \theta_i) - F(s_{n+1} - \theta_i))},$$

$$\hat{p}_j(\overline{a}) = \frac{p_j(1 - F(s_N - \theta_j))}{1 - \sum_{i=1}^{T} p_i F(s_N - \theta_i)}.$$

As the distribution of states is symmetric about 0 and single-peaked, $F(s + |\theta|) + F(s - |\theta|)$ is increasing in $|\theta|$ when $s < 0$, decreasing in $|\theta|$ when $s > 0$, and constant in $|\theta|$ when $s = 0$.

Now note that, for any $j \leq \frac{T-1}{2}$, the probability of a PM who chose $\underline{a}$ having bias at least θ_j in magnitude is

$$\begin{aligned}
\sum_{i=1}^{j} [\hat{p}_i(\underline{a}) + \hat{p}_{T+1-i}(\underline{a})] &= \frac{\sum_{i=1}^{j} [p_i F(s_2 - \theta_i) + p_{T+1-i} F(s_2 - \theta_{T+1-i})]}{\sum_{i=1}^{T} p_i F(s_2 - \theta_i)} \\
&= \frac{\sum_{i=1}^{j} p_i [F(s_2 - \theta_i) + F(s_2 + \theta_i)]}{\sum_{i=1}^{\frac{T-1}{2}} p_i [F(s_2 - \theta_i) + F(s_2 + \theta_i)] + p_{\frac{T+1}{2}} F(s_2)}.
\end{aligned}$$

Hence we can conclude that for all $j \leq \frac{T-1}{2}$,

$$s_2 < 0 \implies \sum_{i=1}^{j} [\hat{p}_i(\underline{a}) + \hat{p}_{T+1-i}(\underline{a})] > \sum_{i=1}^{j} 2p_i,$$

$$s_2 \leq 0 \implies \sum_{i=1}^{j} [\hat{p}_i(\underline{a}) + \hat{p}_{T+1-i}(\underline{a})] \leq \sum_{i=1}^{j} 2p_i.$$

Similarly, for any $j \leq \frac{T-1}{2}$, the probability of a PM who chose $\overline{a}$ having bias at least θ_j in magnitude is

$$\sum_{i=1}^{j} [\hat{p}_i(\overline{a}) + \hat{p}_{T+1-i}(\overline{a})] = \frac{\sum_{i=1}^{j} p_i [(1 - F(s_N - \theta_i)) + (1 - F(s_N + \theta_i))]}{\sum_{i=1}^{\frac{T-1}{2}} p_i [(1 - F(s_N - \theta_i)) + (1 - F(s_N + \theta_i))] + p_{\frac{T+1}{2}} (1 - F(s_N))}.$$

Hence, for all $j \leq \frac{T-1}{2}$,

$$s_N > 0 \implies \sum_{i=1}^{j} [\hat{p}_i(\overline{a}) + \hat{p}_{T+1-i}(\overline{a})] > \sum_{i=1}^{j} 2p_i,$$
$$s_N \leq 0 \implies \sum_{i=1}^{j} [\hat{p}_i(\overline{a}) + \hat{p}_{T+1-i}(\overline{a})] \leq \sum_{i=1}^{j} 2p_j.$$

Assumption 2 and the above calculations imply that any partition with $V(\hat{p}(\overline{a})) = V(\hat{p}(\underline{a}))$ and $s_2 < s_N$ must involve $s_2 < 0 < s_N$. Thus, to verify Condition 2, we can restrict attention to partitions with $s_2 < 0 < s_N$.

Let a_n be an action taken with positive probability for which $0 \in [s_n, s_n + 1]$. Since conditional probabilities are martingales, the previous arguments imply that for all $j \leq \frac{T-1}{2}$,

$$\sum_{i=1}^{j} (\Pr[\theta = \theta_i | a < a_n] + \Pr[\theta = \theta_{T+1-i} | a < a_n]) \geq \sum_{i=1}^{j} 2p_i, \text{ and}$$
$$\sum_{i=1}^{j} (\Pr[\theta = \theta_i | a > a_n] + \Pr[\theta = \theta_{T+1-i} | a > a_n]) \geq \sum_{i=1}^{j} 2p_i,$$

with at least one inequality strict. Hence, the probability of a PM who chose a_n having bias at least $|\theta_j|$ in magnitude is

$$\sum_{i=1}^{j} [\hat{p}_i(a_n) + \hat{p}_{T+1-i}(a_n)] < \sum_{i=1}^{j} 2p_i < \sum_{i=1}^{j} [\hat{p}_i(\overline{a}) + \hat{p}_{T+1-i}(\overline{a})].$$

Hence, by Assumption 2 and Assumption 4, $V(\hat{\boldsymbol{p}}(a_n)) > V(\hat{\boldsymbol{p}}(\overline{a}))$, as required. □

Proof of Proposition 2. Assume the hypotheses. Let $A(\boldsymbol{s})$ denote the set of actions taken on the equilibrium path given $\boldsymbol{s}$. By Assumption 1, Assumption 3 and continuity, there exists $B > 0$ such that $\min_\theta \mathcal{U}(\boldsymbol{\delta_\theta}, 0)$ is higher than the payoff from any $\boldsymbol{s} \notin \mathcal{S}_B^L := \{\boldsymbol{s} \in \mathcal{S}^L : s_N - s_2 \geq B\}$. By Lemma 1 it is sufficient to show that for any symmetric $\boldsymbol{p}$ there exists an $\varepsilon > 0$ such that, for all $\boldsymbol{q}$ with $\max_{\{j \in \{1,\ldots,T\}} |p_j - q_j| < \varepsilon$, there does not exist a $\boldsymbol{s} \in \mathcal{S}_B^L$ such that $V(\hat{\boldsymbol{q}}(a))$ is constant on $A(\boldsymbol{s})$. By Lemma 3 and Condition 2, it follows that

$$d_{\boldsymbol{s}} := \max_{a \in A(\boldsymbol{s})} V(\hat{\boldsymbol{p}}(a)) - \min_{a \in A(\boldsymbol{s})} V(\hat{\boldsymbol{p}}(a)) > 0, \text{ and } d := \min_{s \in \boldsymbol{S}_B^L} d_{\boldsymbol{s}} > 0,$$

where the second part uses the fact that $\mathcal{S}_B^L$ is compact and $V(\cdot)$ is continuous.

Since $V(\cdot)$ is continuous on $\Delta\Theta$ and $\Delta\Theta$ is compact, $V(\cdot)$ is uniformly continuous. Moreover, since the set of priors is compact, for any a, $\hat{\boldsymbol{p}}(a)$ is uniformly continuous in $\boldsymbol{p}$ given $\boldsymbol{s}$. Hence, for any $\boldsymbol{s} \in \mathcal{S}_B^L$ and $a \in A(\boldsymbol{s})$, there exists an $\varepsilon_1(a, \boldsymbol{s}) > 0$ such that $|V(\hat{\boldsymbol{p}}) - V(\hat{\boldsymbol{p}}')| < \frac{d}{2}$ for any priors $\boldsymbol{p}$ and $\boldsymbol{p}'$ for which $\max_{j \in \{1,\ldots,T\}} |p_j - p_j'| < \varepsilon_1(a, \boldsymbol{s})$. Furthermore, since $A(\boldsymbol{s}) \subseteq A$ is finite and $\mathcal{S}_B^L$ is compact,

$$\varepsilon = \min_{\boldsymbol{s} \in \mathcal{S}_B^L} \min_{a \in A(\boldsymbol{s})} \varepsilon_2(a, \boldsymbol{s}) > 0.$$

Consequently, for any $\boldsymbol{q}, \boldsymbol{p}$ such that $\max_{j \in \{1,\ldots,T\}} |q_j - p_j| < \varepsilon$, for all $\boldsymbol{s} \in \mathcal{S}_B^L$ and all $a \in A(\boldsymbol{s})$,

$$|V(\hat{\boldsymbol{q}}(a)) - V(\hat{\boldsymbol{p}}(a))| < \frac{d}{2}.$$

Now by the definition of d, for all $\boldsymbol{s} \in \mathcal{S}_B^L$, there exist $a, a' \in A(\boldsymbol{s})$ such that $V(\hat{\boldsymbol{p}}(a)) - V(\hat{\boldsymbol{p}}(a')) \geq d$. Hence, for any $\boldsymbol{q}$ such that $\max_{j \in \{1,\ldots,T\}} |p_j - q_j| < \varepsilon$,

$$V(\hat{\boldsymbol{q}}(a')) < V(\hat{\boldsymbol{p}}(a')) + \frac{d}{2} \leq V(\hat{\boldsymbol{p}}(a)) - \frac{d}{2} < V(\hat{\boldsymbol{q}}(a)).$$

The result follows from Lemma 1. □

4.2. Proofs for Section 3

Proof of Proposition 3. Let $\theta \neq 0$ be such that $\mathcal{U}(\boldsymbol{\delta_\theta}, 0) > \mathcal{U}(\boldsymbol{p}, k)$. For any $x \in [0, 2]$ define the following announcement strategy: type-0 announces message 0, type θ announces θ with probability $\min\{x, 1\}$, and all other types announce θ with probability $\max\{x - 1, 0\}$. Denote the voter's beliefs after hearing each of the two messages as $\boldsymbol{p}(0, x)$ and $\boldsymbol{p}(\theta, x)$; note that both beliefs are continuous in x. If we can find an x such that $\mathcal{U}(\boldsymbol{p}(\theta, x), k) = \mathcal{U}(\boldsymbol{p}(0, x), k)$, then the voter is indifferent and we have an informative equilibrium where the candidate announcement strategies are characterized by x and the voter randomizes with equal probability regardless of the messages sent.

Since $\mathcal{U}(\boldsymbol{p}, k)$ is continuous in $\boldsymbol{p}$ (owing to a suitable selection of equilibria in the policy-making stage), it follows that $\mathcal{U}(\boldsymbol{p}(\theta, x), k)$ and $\mathcal{U}(\boldsymbol{p}(0, x), k)$ are continuous in x. Furthermore,

$$\lim_{x \to 0} \mathcal{U}(\boldsymbol{p}(\theta, x), k) = \mathcal{U}(\boldsymbol{\delta_\theta}, k) > \mathcal{U}(\boldsymbol{p}, k) = \mathcal{U}(\boldsymbol{p}(0, 0), k).$$

Conversely,

$$\mathcal{U}(\boldsymbol{p}(\theta, 2), k) < \mathcal{U}(\boldsymbol{\delta_0}, k) = \mathcal{U}(\boldsymbol{p}(0, 2), k).$$

By the intermediate value theorem, there exists $x \in (0, 2)$ such that $\mathcal{U}(\boldsymbol{p}(\theta, x), k) = \mathcal{U}(\boldsymbol{p}(0, x), k)$.

$\square$

References

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