Supplementary Appendix for
Opinions as Incentives

Yeon-Koo Che Navin Kartik

August 13, 2009

This supplementary appendix provides some derivations that were omitted from the paper's appendix. New displayed math is numbered in the form (A-x). Displays without the preface "A-" refer to numbering in the paper.

Proof of Proposition 4, Step 1

Here we provide the derivation for

$$\mathcal{A}_{11}(0,p) > \mathcal{A}_1(0,p) = \mathcal{A}_2(0,p) = 0, \tag{23}$$

where

$$\mathcal{A}(\mu,p) = 2\mu(1-\rho)\rho \int_{s \notin S(B(\mu),p)} (s - \overline{s}(B(\mu),p))\gamma(s;\mu)ds + \rho^2 \int_{s \notin S(B(\mu),p)} (s - \overline{s}(B(\mu),p))^2 \gamma(s;\mu)ds. \tag{22}$$

Claim 1. $\mathcal{A}_1(0,p) = 0$.

Proof. Differentiate (22) to get

$$\begin{aligned}\mathcal{A}_1(\mu,p) &= 2\mu\rho(1-\rho)\frac{\partial}{\partial\mu}\int_{s\notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))\gamma(s;\mu)\,ds \\ &\quad +2\rho(1-\rho)\int_{s\notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))\gamma(s;\mu)\,ds \\ &\quad +\rho^2\frac{\partial}{\partial\mu}\int_{s\notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))^2\gamma(s;\mu)\,ds. \end{aligned} \tag{A-1}$$

Let us evaluate this expression at $\mu = 0$. The first term is obviously 0. The second term is also 0 because $B(0) = 0$ and $S(0,p)$ is a measure zero set (there is full disclosure when $\mu = 0$). To see that the third term is also 0, observe that

$$
\begin{aligned}
& \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s;\mu)\, ds \\
= & \frac{\partial}{\partial \mu} \left\{ \int_{s \in \mathbb{R}} (s - \bar{s}(B(\mu), p))^2 \gamma(s;\mu)\, ds - \int_{s \in S(B(\mu),p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s;\mu)\, ds \right\} \\
= & \frac{\partial}{\partial \mu} \left\{ \mu^2 + \sigma_1^2 + \sigma_0^2 - 2\mu \bar{s}(B(\mu), p) + (\bar{s}(B(\mu), p))^2 - \int_{\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s;\mu)\, ds \right\} \\
= & 2\mu - 2\bar{s}(B(\mu), p) + 2\bar{s}(B(\mu), p)\, \bar{s}_1(B(\mu), p)\, B'(\mu) \\
& + \int_{\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} 2(s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \\
& + \left(\bar{s}_1(B(\mu), p)\, B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \left(-2\frac{B(\mu)}{\rho} \right)^2 \gamma\left(\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; \mu \right),
\end{aligned}
$$

which is 0 when $\mu = 0$, because $B(0) = 0$ and $\bar{s}(0, p) = 0$. ▮

Claim 2. $\mathcal{A}_{11}(0, p) > 0$.

Proof. Differentiating (A-1) yields

$$
\begin{aligned}
\mathcal{A}_{11}(\mu, p) = & \; 2\rho(1-\rho) \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \\
& + 2\mu\rho(1-\rho) \frac{\partial^2}{\partial \mu^2} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \\
& + 2\rho(1-\rho) \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \\
& + \rho^2 \frac{\partial^2}{\partial \mu^2} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s;\mu)\, ds. \qquad \text{(A-2)}
\end{aligned}
$$

Let us evaluate this expression at $\mu = 0$. The second term is obviously 0. For the first and third terms, we have

$$
\begin{aligned}
& \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \\
= & \frac{\partial}{\partial \mu} \left\{ \int_{s \in \mathbb{R}} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds - \int_{s \in S(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \right\} \\
= & \frac{\partial}{\partial \mu} \left\{ \mu - \bar{s}(B(\mu), p) - \int_{\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} (s - \bar{s}(B(\mu), p))\, \gamma(s;\mu)\, ds \right\} \\
= & 1 - \bar{s}_1(B(\mu), p)\, B'(\mu) + \int_{\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} \bar{s}_1(B(\mu), p)\, B'(\mu)\, \gamma(s;\mu)\, ds \\
& + \left(\bar{s}_1(B(\mu), p)\, B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \left(-2\frac{B(\mu)}{\rho} \right) \gamma\left(\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; \mu \right),
\end{aligned}
$$

which is equal to 1 at $\mu = 0$ because $\bar{s}_1(0,p) = 0$ (Step 3 in proof of Proposition 1).

Finally, for the fourth term in (A-2), we have

$$\begin{aligned}
&\frac{\partial^2}{\partial \mu^2} \int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))^2 \gamma(s;\mu)\, ds \\
= \; & \frac{\partial}{\partial \mu} \left\{ \begin{array}{l} 2\mu - 2\bar{s}(B(\mu),p) + 2\bar{s}(B(\mu),p)\,\bar{s}_1(B(\mu),p)\,B'(\mu) \\ +2\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds \\ +\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\left(-2\frac{B(\mu)}{\rho}\right)^2 \gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};\mu\right) \end{array} \right\} \\
= \; & 2\left(1 - \bar{s}_1(B(\mu),p)\,B'(\mu) + B'(\mu)\left(\bar{s}(B(\mu),p)\,\bar{s}_{11}(B(\mu),p)\,B'(\mu) + (\bar{s}_1(B(\mu),p))^2 B'(\mu)\right)\right) \\
& -2\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} \bar{s}_1(B(\mu),p)\,B'(\mu)\,\gamma(s;\mu)\,ds \\
& -2\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\left(-2\frac{B(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};\mu\right) ds \\
& +\frac{4}{\rho^2} 2B(\mu)\,B'(\mu)\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};\mu\right) \\
& +\left(-2\frac{B(\mu)}{\rho}\right)^2 \frac{\partial}{\partial \mu}\left\{\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};\mu\right)\right\},
\end{aligned}$$

which is equal to 2 at $\mu = 0$ because $B(0) = \bar{s}(0,p) = \bar{s}_1(0,p) = 0$.

Since $\rho \in (0,1)$, it follows that (A-2) is strictly positive, as desired. ▌

Claim 3. $\mathcal{A}_2(0,p) = 0$.

Proof. Differentiate (22) to get

$$\begin{aligned}
\mathcal{A}_1(\mu,p) \;=\; & 2\mu\rho(1-\rho)\frac{\partial}{\partial p}\int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds \\
& +\rho^2 \frac{\partial}{\partial p}\int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))^2\,\gamma(s;\mu)\,ds. \qquad \text{(A-3)}
\end{aligned}$$

Let us evaluate this at $\mu = 0$. To see that the first term is 0, observe that

$$\begin{aligned}
&\frac{\partial}{\partial p}\int_{s \notin S(B(\mu),p)} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds \\
= \; & \frac{\partial}{\partial p}\left\{\int_{s\in\mathbb{R}} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds - \int_{s \in S(B(\mu),p)} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds\right\} \\
= \; & \frac{\partial}{\partial p}\left\{\mu - \bar{s}(B(\mu),p) - \int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} (s - \bar{s}(B(\mu),p))\,\gamma(s;\mu)\,ds\right\} \\
= \; & -\bar{s}_2(B(\mu),p) - \int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} (-\bar{s}_2(B(\mu),p))\,\gamma(s;\mu)\,ds \\
& +(\bar{s}_2(B(\mu),p))\left(-2\frac{B(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};\mu\right),
\end{aligned}$$

which is equal to $-\bar{s}_2(0,p)$ when $\mu=0$ because $B(0)=\bar{s}(0,p)=0$; and in turn, $\bar{s}_2(0,p)=0$ since $\bar{s}(0,p)=0$ for all p.

To see that the second term in (A-3) is 0 at $\mu=0$, observe that

$$\begin{aligned}
&\frac{\partial}{\partial p}\int_{s\notin S(B(\mu),p)}(s-\bar{s}(B(\mu),p))^2\gamma(s;\mu)\,ds \\
=\;&\frac{\partial}{\partial p}\left\{\mu^2+\sigma_1^2+\sigma_0^2-2\mu\bar{s}(B(\mu),p)+(\bar{s}(B(\mu),p))^2-\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}(s-\bar{s}(B(\mu),p))^2\gamma(s;\mu)\,ds\right\} \\
=\;&2\mu\bar{s}_2(B(\mu),p)+2(\bar{s}(B(\mu),p))\bar{s}_2(B(\mu),p) \\
&-\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}2(s-\bar{s}(B(\mu),p))(-\bar{s}_2(B(\mu),p))\gamma(s;\mu)\,ds \\
&+(\bar{s}_2(B(\mu),p))\left(-2\frac{B(\mu)}{\rho}\right)^2\gamma\left(\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho};\mu\right),
\end{aligned}$$

which is 0 at $\mu=0$, because $B(0)=\bar{s}(B(0),p)=0$. ∎

Proof of Proposition 4, Step 2

Here we verify that

$$v_1(0,p)=v_2(0,p)=v_{11}(0,p)=w_1(0,p)=w_2(0,p)=w_{11}(0,p)=0, \tag{25}$$

where we had defined

$$w(\mu,p):=-\tilde{\sigma}^2-\int_{\underline{s}(B(\mu),p)}^{\bar{s}(B(\mu),p)}(a_\emptyset(B(\mu),p)-s\rho)^2\gamma(s;0)ds,$$

and

$$v(\mu,p):=-\tilde{\sigma}^2-\int_{-\infty}^{\infty}(a_\emptyset(B(\mu),p)-s\rho)^2\gamma(s;0)ds.$$

First, since $a_\emptyset(B(\mu),p)=\rho\bar{s}(B(\mu),p)$,

$$\begin{aligned}
v_1(\mu,p) &= -\int_{-\infty}^{\infty}2(\rho\bar{s}(B(\mu),p)-s\rho)\rho\bar{s}_1(B(\mu),p)B'(\mu)\gamma(s;0)ds, \qquad \text{(A-4)}\\
v_2(\mu,p) &= -\int_{-\infty}^{\infty}2(\rho\bar{s}(B(\mu),p)-s\rho)\rho\bar{s}_2(B(\mu),p)\gamma(s;0)ds.
\end{aligned}$$

Since $B(0)=\bar{s}_1(0,p)=\bar{s}_2(0,p)=0$, it follows that $v_1(0,p)=v_2(0,p)=0$.

Second, noting also that $\underline{s}(B(\mu),p) = \bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho}$,

$$
\begin{aligned}
w_1(\mu,p) &= -\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} 2\left(\rho\bar{s}(B(\mu),p) - \rho s\right)\rho\bar{s}_1(B(\mu),p)\,B'(\mu)\,\gamma(s;0)ds \\
&\quad + \left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\left(-2\frac{B(\mu)}{\rho}\right)^2 \gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};0\right), \quad \text{(A-5)} \\
w_2(\mu,p) &= -\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)} 2\left(\rho\bar{s}(B(\mu),p) - \rho s\right)\rho\bar{s}_2(B(\mu),p)\,\gamma(s;0)ds \\
&\quad + \bar{s}_2(B(\mu),p)\left(-2\frac{B(\mu)}{\rho}\right)^2 \gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};0\right).
\end{aligned}
$$

Since $B(0) = \bar{s}_1(0,p) = \bar{s}_2(0,p) = 0$, it follows that $w_1(0,p) = w_2(0,p) = 0$.

Third, differentiating (A-4) yields

$$
\begin{aligned}
v_{11}(\mu,p) &= -2\rho^2(1-\rho)\frac{\partial}{\partial\mu}\int_{-\infty}^{\infty}\left(\bar{s}(B(\mu),p) - s\right)\bar{s}_1(B(\mu),p)\,\gamma(s;0)ds \\
&\propto -\bar{s}_{11}(B(\mu),p)\int_{-\infty}^{\infty}\left(\bar{s}(B(\mu),p) - s\right)\gamma(s;0)ds - (1-\rho)\int_{-\infty}^{\infty}\left(\bar{s}_1(B(\mu),p)\right)^2\gamma(s;0)ds, \quad \text{(A-6)}
\end{aligned}
$$

where the second line has ignored the factor $2\rho^2(1-\rho)$. Let us evaluate (A-6) at $\mu = 0$: since $B(0) = \bar{s}(0,p) = 0$, the first integral term is $\int_{-\infty}^{\infty} s\gamma(s;0)ds = 0$, while the second integral term is also 0 since $\bar{s}_1(0,p) = 0$. Therefore, $v_{11}(0,p) = 0$.

Fourth, differentiating (A-5) yields

$$
\begin{aligned}
w_{11}(\mu,p) &= -2\rho^2(1-\rho)\,\bar{s}_{11}(B(\mu),p)\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}\left(\bar{s}(B(\mu),p) - s\right)\gamma(s;0)ds \\
&\quad -2\rho^2(1-\rho)^2\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}\left(\bar{s}_1(B(\mu),p)\right)^2\gamma(s;0)ds \\
&\quad +\frac{\partial}{\partial\mu}\left[\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\left(-2\frac{B(\mu)}{\rho}\right)^2\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};0\right)\right] \\
&= -2\rho^2(1-\rho)\,\bar{s}_{11}(B(\mu),p)\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}\left(\bar{s}(B(\mu),p) - s\right)\gamma(s;0)ds \\
&\quad -2\rho^2(1-\rho)^2\int_{\bar{s}(B(\mu),p)-2\frac{B(\mu)}{\rho}}^{\bar{s}(B(\mu),p)}\left(\bar{s}_1(B(\mu),p)\right)^2\gamma(s;0)ds \\
&\quad +\frac{4}{\rho^2}2B(\mu)\,B'(\mu)\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - 2\frac{B'(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};0\right) \\
&\quad +\left(\frac{-2B(\mu)}{\rho}\right)^2\frac{\partial}{\partial\mu}\left[\left(\bar{s}_1(B(\mu),p)\,B'(\mu) - \frac{2B'(\mu)}{\rho}\right)\gamma\left(\bar{s}(B(\mu),p) - 2\frac{B(\mu)}{\rho};0\right)\right] \quad \text{(A-7)}
\end{aligned}
$$

Let us evaluate (A-7) at $\mu = 0$. Since $B(0) = \bar{s}(0,p) = 0$, the first two terms have integrals over measure zero sets, hence are 0. Moreover, $B(0) = 0$ implies that the second two terms are also 0. Therefore, $w_{11}(0,p) = 0$.

Proof of Proposition 8

Here we show that $g'(0) = 0$ while $g''(0) = p''(0)(\sigma_0^2 - \tilde{\sigma}^2)(\lambda - (f'(0))^2)$, where

$$g(\mu) := \lambda\left(-p(\mu)\tilde{\sigma}^2 - (1-p(\mu))\sigma_0^2\right) - c(p(\mu) + (p(\mu) - p(f(\mu)))(\sigma_0^2 - \tilde{\sigma}^2 + \mu^2 - (B(\mu))^2).$$

Taking the first derivative yields

$$\begin{aligned} g'(\mu) &= \lambda\left(\sigma_0^2 - \tilde{\sigma}^2\right)p'(\mu) - c'(p(\mu))\,p'(\mu) + \left(\sigma_0^2 - \tilde{\sigma}^2\right)\left[p'(\mu) - p'(f(\mu))\,f'(\mu)\right] \\ &\quad + \left(p'(\mu) - p'(f(\mu))\,f'(\mu)\right)\left(\mu^2 - B(\mu)^2\right) + (p(\mu) - p(f(\mu)))\,2\left(\mu - B(\mu)(1-\rho)\right), \end{aligned}$$

and because $p'(0) = 0 = f(0)$, we have $g'(0) = 0$.

Taking the second derivative yields

$$\begin{aligned} g''(\mu) &= \lambda\left(\sigma_0^2 - \tilde{\sigma}^2\right)p''(\mu) - p''(\mu)\,c'(p(\mu)) - c''(p(\mu))\left(p'(\mu)\right)^2 \\ &\quad + \left(\sigma_0^2 - \tilde{\sigma}^2\right)\left[p''(\mu) - p''(f(\mu))\left(f'(\mu)\right)^2 - p'(f(\mu))\,f''(\mu)\right] \\ &\quad + \left[p''(\mu) - p''(f(\mu))\left(f'(\mu)\right)^2 - p'(f(\mu))\,f''(\mu)\right]\left(\mu^2 - (B(\mu))^2\right) \\ &\quad + \left(p'(\mu) - p'(f(\mu))\,f'(\mu)\right)2\left(\mu - B(\mu)(1-\rho)\right) + \left[p'(\mu) - p'(f(\mu))\,f'(\mu)\right]2\left(\mu - B(\mu)(1-\rho)\right) \\ &\quad + (p(\mu) - p(f(\mu)))\,2\left(1 - (1-\rho)^2\right). \end{aligned}$$

Evaluating at $\mu = 0$, and using the facts that $p'(0) = 0 = f(0)$, we get

$$g''(0) = \lambda\left(\sigma_0^2 - \tilde{\sigma}^2\right)p''(0) - p''(0)\,c'(p(0)) + \left(\sigma_0^2 - \tilde{\sigma}^2\right)\left[p''(0) - p''(0)\left(f'(0)\right)^2\right],$$

and now using the fact that $c'(p(0)) = \sigma_0^2 - \tilde{\sigma}^2$ (by the first-order condition for optimality of $p(0)$), we further simplify to

$$\begin{aligned} g''(0) &= \lambda\left(\sigma_0^2 - \tilde{\sigma}^2\right)p''(0) - p''(0)\left(\sigma_0^2 - \tilde{\sigma}^2\right) + \left(\sigma_0^2 - \tilde{\sigma}^2\right)p''(0) - \left(\sigma_0^2 - \tilde{\sigma}^2\right)p''(0)\left(f'(0)\right)^2 \\ &= p''(0)\left(\sigma_0^2 - \tilde{\sigma}^2\right)\left(\lambda - \left(f'(0)\right)^2\right). \end{aligned}$$